Minor Loops Calculation with a Modified Jiles-Atherton Hysteresis Model

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Abstract—This work proposes a modification in the Jiles-Atherton hysteresis model in order to improve the minor loops representation. The irreversible magnetization component is slightly modified keeping unchanged the other model equations and the model simplicity. Differently to other proposed methodologies found in the literature, the previously knowledge of the magnetic field waveform is not need to assure closed minor loops. Measured and calculated hysteresis curves are used in order to validate the methodology.

Index Terms—Hysteresis models, magnetic materials, minor loops.

I. INTRODUCTION

An increasing harmonic content can be observed in electrical systems. Odd harmonics are frequently found in rotating electrical machines. The third-harmonic flux, for instance, exists due to the saturation of the machines iron core. Moreover non-sinusoidal electrical sources, in particular PWM (Pulse Width Modulation) converters are now the supply standard option for rotating machines and electromechanical devices. These converters contribute to the increase of the harmonic content in electrical systems and then to the increase of iron losses.

The increase of copper losses can be modeled in the windings with the additional harmonic content but the magnetic core hysteresis losses under a PWM voltage is a more complex process with a strongly non-linear material behavior [1].

The harmonic content causes a distortion in the flux waveform deviating it from the sinusoidal one. The distorted flux waveform can add minor loops to the major hysteresis loop which increases the core losses. By minor loops one can consider any closed B-H symmetric or asymmetric curve, other than the saturated one. The empirical losses models as, for instance, the Steinmetz model are not adequate to take into account minor loops, so a hysteresis model must be employed.

Nowadays, research in hysteresis modeling is mainly focused on the development of history-dependent models. These ones must present closed loops which must be immediately stable [2].

Preisach’s hysteresis model is a mathematically based formulation widely employed for modeling the behavior of magnetic materials [3]. It can to represent with good accuracy the centered major loop and also not-centered minor loops. In fact, the history of the material magnetic state is an intrinsic
property of this model. However, the numerical implementation of the Preisach’s model and its parameters identification require more effort in comparison with some other models such as the Jiles-Atherton (JA) model [4].

Among the proposed hysteresis models in recent years, the JA has been one of the most investigated [5]. The classical JA model is based on physical assumptions. It uses a magnetic energy balance in the material leading to a first order differential equation with five parameters to be identified. Nevertheless, this differential equation is not able to reproduce properly minor loops. In fact, the JA model presents a slow accommodation time, so the magnetization trajectory between the turning points of a minor loop will not be closed at the end of its excursion. This remains the major limitation in the use of the JA model.

In order to overcome this drawback, some works have been proposed. In [6] a modification in the model was proposed but it requires a priori the knowledge of the magnetic field evolution. Other works propose the use of scaling factors in the main equation of the model [7] or the use of different parameters set to represent the major and minor loops [8][9].

In this work, we present a generalized JA model able to predict the core losses in the material under sinusoidal or distorted waveform fluxes. The resultant model is based in a combination of theoretical and empirical approaches and extensive experimental observation. No major modifications in the model are needed. The proposed technique improves the JA model allowing it to represent non-centered minor loops. Contrarily to other modifications of the original model, the previous knowledge of the magnetic field waveform is not necessary.

Calculated and measured results will be compared in order to validate the proposed methodology.

II. THE SCALAR JILES-ATHERTON HYSTERESIS MODEL

In the original JA model, the magnetization $M$ is decomposed into its reversible $M_{rev}$ and irreversible $M_{irr}$ components [5]. The relationship linking these contributions is given by:

$$M_{rev} = c(M_{an} - M_{irr})$$

The anhysteretic magnetization $M_{an}$ is given by the Langevin function:

$$M_{an} = M_s \left[ \coth \left( \frac{H_e}{a} \right) - \frac{a}{H_e} \right]$$

with:

$$\frac{dM_{irr}}{dH_e} = \frac{M_{an} - M_{irr}}{k\delta}$$
where \( H_e = H + \alpha M \) is the effective field.

\( M_S, \alpha, \alpha, c \) and \( k \) are the model parameters which can be obtained from measured hysteresis loops; \( \delta \) is a directional parameter assuming the value +1 if \( dH/dt > 0 \) and -1 if \( dH/dt < 0 \).

Combining the equations above the main equation of the original JA model can be written as:

\[
\frac{dM}{dH} = \frac{(1-c) \frac{dM_{irr}}{dHe} + c \frac{dM_{an}}{dHe}}{1 - \alpha (1-c) \frac{dM_{irr}}{dHe} - \alpha c \frac{dM_{an}}{dHe}}
\]

The differential equation (4) allows calculating the magnetization \( M \) with respect to \( H \) variations. However in some applications the induction \( B \) is known prior to the field. The finite element method with magnetic potential formulation is an example of such applications. In [10] an inverse model where the independent variable is the induction \( B \) was presented. The main equation of this inverse model is:

\[
\frac{dM}{dB} = \frac{(1-c) \frac{dM_{irr}}{dB_e} + c \frac{dM_{an}}{dB_e}}{1 + \mu_0 (1-\alpha) (1-c) \frac{dM_{irr}}{dB_e} + c (1-\alpha) \frac{dM_{an}}{dB_e}}
\]

where:

\[
\frac{dM_{irr}}{dB_e} = \frac{M_{an} - M_{irr}}{\mu_0 k \delta}
\]

and \( B_e = \mu_0 H_e \) is the effective magnetic flux density.

III. MINOR LOOP REPRESENTATION WITH JILES-ATHERTON MODEL

Although the original JA model is able to represent a wide range of major hysteresis loops, in particular those of soft magnetic materials, it can produces non-physical minor loops with its classical equations.

An example of such non consistent curve is the 1-2-3 minor loop shown in Fig. 1. The minor loop starts in the reversal point 1, reaches the point 2 but does not return to the initial point memory, instead, goes to a new point 3.

Being the magnetization, in the JA model, only dependant of its previous time step value and the solution of the differential equation (4) or (5), there is no condition or rule that guarantees the return to the initial point memory.
In fact, the inconsistency resides in the slow accommodation time in the differential equation, especially in the high slope curves. With a symmetrical high amplitude excitation the accommodation time is enough to produce a closed major loop. On the other hand, after a perturbation in the magnetization excursion (a turning point), it spends some calculation time to recover a stable trajectory.

IV. PROPOSED METHODOLOGY

One can observe in the classical model that the closure of minor loops is not guaranteed due a strong variation rate of the global magnetization. When running through a minor loop, we observe a transitory in the trajectory of magnetization that requires a time to recover its stability. As the minor loop finishes when the magnetization passes by its first turning point (point 3 is far from point 1 in Fig. 1) the accommodation time is not reached.

If one considers that running through minor loops yields to a small perturbation of the magnetization around a given material magnetic state, it would be natural to limit the variation rates of the magnetization in order to allow the stability to be achieved.

In the proposed methodology the first step consists of recognizing the magnetization running through a minor loop. With the JA model, one can identify the minor loop turning points just observing the directional parameter \( \delta \). It indicates if the magnetization is on its ascendant \( (\delta = +1) \) or descendent \( (\delta = -1) \) hysteresis branch. A change in \( \delta \) value indicates that a turning point occurred.

Storing the \( B \) and \( H \) values in the two consecutives turning points we can estimate if the magnetization is running through an asymmetric minor loop or a centered curve.

Observing numerical results on can see that the first portion of the calculated minor loop has a convenient behavior so the procedure here proposed is applied after the second turning point.

As the global magnetization is mainly irreversible in the JA model one can choose to perform a modification on the inverse magnetization component. The variation rate of (3) or (6) is weighted by a continuous function. The equation (6) in the inverse model now will be calculated as:
\[
\frac{dM_{ir}}{dB_c} = w(H) \frac{M_{an} - M_{ir}}{\mu_0 k \delta}.
\]  

\text{(7)}

with, for example, a sigmoid function:

\[
w(H) = \frac{1}{1 + e^{-\frac{|H| - \beta}{\eta}}}
\]

\text{(8)}

where \(\beta\) and \(\eta\) are parameters to be obtained from experimental data.

The weight factor is applied locally and becomes 1 once the minor loop is closed, \(i.e.,\), the magnetization is close enough to that of first turning point. This rule can approximate the model behavior to the experimental observation. However the return to the initial turning point is rather a tendency than a rigorous mathematical rule.

Additionally, some adjustments can be performed on the model parameters to improve the minor loop representation.

V. RESULTS

A voltage fed Epstein frame was chosen to verify experimentally the proposed methodology. A 0.5 mm, non-oriented, Fe-Si 3% sheet was used. The flux was controlled imposing the voltage in the secondary winding and the current was directly measured in the primary. The Labview\textsuperscript{TM} system was used to control the induction and to supply several distorted waveforms.

The inverse model was used to perform the calculations. The five model parameters (Table I) were obtained with a fitting procedure based on genetic algorithms [11].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
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<tbody>
<tr>
<td>(M_S)</td>
<td>(1.4817 \times 10^6 \text{ [A/m]})</td>
</tr>
<tr>
<td>(k)</td>
<td>(6.5533 \times 10 \text{ [A/m]})</td>
</tr>
<tr>
<td>(c)</td>
<td>(3.1632 \times 10^{-1})</td>
</tr>
<tr>
<td>(a)</td>
<td>(7.1330 \times 10 \text{ [A/m]})</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>(1.4717 \times 10^{-3})</td>
</tr>
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Table I. Parameters of IA model.

Fig. 2 shows a measured hysteresis loop obtained with a fundamental frequency of 1 Hz plus its 3\textsupers} and 5\textsupers harmonics.
The measured $B$ curve was used as the model entry. Fig. 3 shows the hysteresis loop calculated with the original model equations.

Fig. 3 illustrates the limitation of the JA model in presence of minor loops. This non-physical behaviour limits the model employment in the magnetic calculations and circuit analysis under a harmonic excitation. Furthermore, the magnetic loss per cycle in the measured and calculated curves above presents an error close to 20%.

The calculation with the proposed technique is shown in Fig. 4. The used weighting function is a sigmoid (8) with $\beta = 54.976$ and $\eta = 45.013$. Also, the parameter $a$, connected with the slope of Langevin function, was adjusted to improve the resultant minor loop. If the weighting function is different from 1, parameter $a$ is considered equal to $8.7735 \times 10^1$ [A/m] and recovers its original value once the minor loop is closed.

The minor loops in this calculation have a good agreement with the measured ones. The difference between measured and calculated losses is now about 3%.
VI. CONCLUSION

In order to overcome the drawback of non closure of minor loops, a modification in the Jiles-Atherton model was proposed.

The limitation in the irreversible magnetization rates with a sigmoid function, associated with some parameter adjustments, allowed a more adequate model behavior when running through minor loops. The possibility of using other weight functions is in the offering. The proposed technique maintains the JA model simplicity being an alternative to the Preisach model implementation, for instance in a finite element code.

The agreement between simulated and measured results demonstrates the methodology efficiency in the case of Fe-Si iron steel. The model can be suitably incorporated into a transient circuit simulator taking into account ferromagnetic materials.

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REFERENCES