Non-resonant Permittivity Measurement Methods

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Abstract— The measurement of the dielectric properties of materials has been applied in non-destructive tests, humidity measurement, soil analysis and even cancer detection. The methods have been developed for over 70 years based on the interaction of the electromagnetic waves with the material under test. This work presents a general model of scattering parameters for non-resonant methods of transmission/reflection and single-port reflection. Equations for determining permittivity are obtained. New equations for short-circuited load and coupled load in the double reflection method are presented.

Index Terms— Microwave measurements, permittivity, short-circuit transmission line method, transmission/reflection method.

I. INTRODUCTION

The dielectric properties characterization is fundamental in engineering. This is employed in non-destructive test and evaluation [1], moisture measurements [2], soil analysis and tumor tissue detection. The physical concepts and technological aspects are related to determining the dielectric properties from the interaction of the electromagnetic fields with the material. These fields must be generated, guided or radiated over the sample (MUT – material under test) and detected after the interaction. Traditionally, these tasks were performed using microwave instrumentation techniques in laboratory [3]. Simultaneously, measurements methods [4] and mathematical methods for propagation, radiation and scattering of microwaves were developed [5]. These methods can be divided in resonant and non-resonant. The non-resonant methods are suitable for broadband measurement. Among these methods the most important ones are the SCTL (short-circuit transmission line) [3] and the NRW (Nicholson-Ross-Weir) [6][7]. The purpose of this work is to generate explicit equations for the permittivity using a straightforward scattering parameters model for load-terminated samples.

II. PERMITTIVITY MEASUREMENT METHODS

A. Historical Development

In 1946, Roberts and Von Hippel [8] presented a method for the measurement of permittivity using an air-filled rectangular waveguide with a sample of MUT in the end of the waveguide.
comparing the standing wave pattern of the partially sample-filled waveguide and that of a short-terminated air-filled waveguide it is possible to determine the permittivity. Such method is known as SCTL reflection method. It obtains the line impedance from the peaks and valleys of the voltage standing wave pattern. This method was still widely used in 1961, when [9] reports uncertainties of 2% for the permittivity and 5% for the loss tangent. The use of charts for hyperbolic functions was avoided by having sample lengths of $\frac{1}{4}$ of the wavelength inside the material. In 1974, a computer program was developed aiming to increase the precision of the Roberts-von Hippel method [10].

In 1970, Nicolson and Ross [6], using a sampling oscilloscope, a sub nanosecond pulse generator and the Fourier transform, obtained the scattering parameters of a sample. With $S_{11}$ and $S_{21}$, expressed as functions of the reflection coefficient in the material-air interface and the transmission coefficient between two faces of the sample, and measured by the aforementioned setup, they obtained the permittivity and permeability of the material. In 1974, Weir [7] obtained the scattering parameters directly from the frequency domain by using an automatic network analyzer, solving the phase ambiguity generated by larger than half wavelength sample length and measuring the group delays in different frequencies, assuming that the permittivity does not change significantly for small variations in frequency. In [11], the problems of the method in dispersive materials are discussed. Regardless of these problems, the method is widely accepted and known as Nicholson-Ross-Weir (NRW) algorithm.

An explicit equation for the permittivity as a function of the transmission and reflection parameters is presented in [12]. The authors show that it is possible to obtain the uncertainty of the permittivity as a function of the sample length, with the lowest uncertainty being obtained when the sample length is a quarter of the wavelength inside the material. The method becomes unstable when the sample length is a multiple of half wavelength.

The resonant methods are inadequate for characterization in the frequency domain. The reflection methods, also known as single-port methods, which measure the reflection coefficient of a guided wave or a wave in free-space [13], can be used for spectroscopy. In [14] such methods are reviewed and possible configurations for the measurements are presented. Among these, the method with two arbitrary terminations can be highlighted. In the reflection methods, the explicit equations for the permittivity are obtained through two or more measurements in two different configurations, as it is shown in [15].

B. Transmission-Reflection methods state-of-the-art

The work in simultaneous measurement of transmission and reflection coefficients of a sample to obtain permittivity is consolidated in [16], in which explicit equations independent of reference plane or sample length are shown. The half wavelength uncertainty is discussed and the measurement uncertainties are determined. Works aiming to solve the half wavelength problem are also referenced. In [17] a new method to solve problems related to dispersive materials is presented. A complete
review regarding the transmission-reflection methods is also done in [17].

C. Single-port reflection methods

Reflection methods which employ the measurement of two reflection coefficients were already presented in [3]. These use which uses a short-terminated transmission line (as the SCTL method) and an open-circuit terminated transmission line. Although the equation for the permittivity is simple [14], the method only works at specific frequencies since, to create an open circuit, it is necessary to create a short-circuit at a quarter-wavelength distance. In [15] an explicit equation with the $S_{11}$ parameter (measured with a coupled load or free-space and a short-circuit) is shown. Other approach is described in [18], using only the amplitude of the reflection coefficient. Two distant frequencies (in non-dispersive media) or three near frequencies (in dispersive media) can be used. The simplicity of the required instrumentation makes the method very attractive.

III. DIELECTRIC SLAB SCATTERING PARAMETERS MODEL

A. Reflection coefficient $\Gamma$ and propagation factor $T$

Consider an uniform dielectric slab, with complex permittivity “$\varepsilon_2$” and thickness “$d$” immersed in a dielectric with permittivity $\varepsilon_1$ to the left and $\varepsilon_3$ to the right, splitting the space into regions 1 and 3, as shown in Fig. 1.

Let us assume an electromagnetic wave, which is perpendicularly incident at the interface ($z=-d$). The incident electric and magnetic waves at the interface are $E_1^+$ and $H_1^+$, respectively. Both are parallel to the interface and are partially reflected to the medium 1 and partially transmitted to the interior of the slab. $E_1^-$ and $H_1^-$ are the reflected waves, which travel in the medium 1 in the negative $z$ direction. From $z=-d$, the transmitted waves $E_2^+$ and $H_2^+$ travel in the positive $z$ direction. On the interface between the slab and the medium 3 ($z=0$) there are the fields $E_{20}^+$ and $H_{20}^+$. These fields are partially transmitted to medium 3, indicated as waves $E_3^+$ and $H_3^+$ and partially reflected back to medium 2, the waves $E_{20}^-$ and $H_{20}^-$. The propagation constants of the materials are $\gamma_1$, $\gamma_2$ e $\gamma_3$. The electric and magnetic fields are related in each medium by the intrinsic impedance of these media: $\eta_1$, $\eta_2$, and $\eta_3$. The transmitting and receiving coefficients in a dielectric slab are $S_{11}$ and $S_{21}$, respectively.
If the medium 3 is infinite, there will be no propagation in the negative \( z \) direction in this medium (no reflection) and \( E_z = 0 \). If medium 1 is vacuum and medium 3 has the same permittivity of medium 2, the value of \( \Gamma \), the reflection coefficient at the interface, is given by:

\[
\frac{E_i}{E_i^+} = \sqrt{\frac{r}{r+1}}
\]

where \( \mu_r \) and \( \epsilon_r \) are the relative permeability and permittivity of medium 2, respectively.

If the media have the magnetic permeability of vacuum (\( \mu_r = 1 \)), the coefficient is simplified to:

\[
\frac{E_i}{E_i^+} = \frac{1}{1+\sqrt{r}}
\]

The propagation constant \( \gamma \) in a dielectric with negligible conductivity and with magnetic permeability equal to the vacuum can be approximated to:

\[
\gamma = \sqrt{\frac{\epsilon}{c}}
\]

where \( c \) is the velocity of light in the vacuum. Therefore, the propagation of a TEM (transversal electromagnetic) wave through the distance \( d \) in a material with the propagation constant \( \gamma \) can be expressed by the propagation factor \( T \) [7]. Using (3) is possible to define:

\[
T = e^{-\gamma d} = e^{-\frac{j\omega}{c}\sqrt{\epsilon} d}
\]

Some authors call \( T \) the “transmission coefficient” [4]. To avoid confusion with the transmission coefficient through a slab, the original term “propagation factor” will be kept.

**B. MUT (Material Under Test) scattering parameters.**

The intrinsic impedance variation between the two different media will result in that part of the incident wave to be transmitted and part of it to be reflected. In a dielectric slab, as shown in *Fig. 2*, there are two interfaces and then multiple reflections will happen inside the slab. Using harmonic analysis, this is simplified in the case of a high loss sample, because the multiple reflections add up to an attenuated standing wave pattern.

The total fields can be obtained through the complete solution of the wave equation inside the slab.
when the dielectric characteristics of the material and the sample dimensions are known. The total field, for \(0 > z > -d\), is:

\[
E_2^t(z) = E_{20}^t e^{-gz} + E_{20}^- e^{gz}
\]  

(5)

The reflection coefficient at the interface between the two media, when the sample is infinite, allows for the substitution of \(E_{20}^-\) through \(\Gamma\) since:

\[
\Gamma = E_1^- - E_1^+ = -E_{20}^- + E_{20}^+
\]  

(6)

The same procedure is applicable to the magnetic field. The total magnetic field can be written as:

\[
H_2^t(z) = H_{20}^- e^{-gz} + H_{20}^+ e^{gz}
\]  

(7)

Since the magnetic fields are related to the electric fields through the intrinsic impedance of the medium, (8) can be rewritten as:

\[
H_2^t(z) = \frac{E_{20}^t}{2} e^{-gz} + \frac{E_{20}^-}{2} e^{gz}
\]  

(9)
Since the media 1 and 3 have the same intrinsic impedance, a scattering parameters model can be applied.

Therefore, the reflection coefficient of the slab, as seen by the incident wave (input), will be the $S_{11}$ parameter itself, or:

$$ S_{11} = \frac{E_1}{E_i} $$

(17)

By expressing $E_i$ as $S_{11}E_i^*$ and dividing (16) by (15):

$$ \frac{1}{(1 + S_{11})} = \frac{1}{2} \left( \frac{T^1 + T}{T^1} \right) $$

(18)

If $\eta_1 = \eta_3 = \eta_0$ and the medium 2 is non-magnetic, the ratio $\eta_1/\eta_2$ is equal to the square root of the relative dielectric permittivity of the medium 2. From (2), is possible to isolate this square root as a function of $\Gamma$ and then obtain the reflection coefficient at the input of the slab:

$$ S_{11} = \left( \frac{1}{1 + 2\tau^2} \right) $$

(19)

The relation between the incident electromagnetic wave at the interface at $z=-d$ and the emerging wave at the interface at $z=0$, when the medium 3 is equal to the medium 1 is the parameter $S_{21}$ itself:

$$ S_{21} = \frac{E_i^*}{E_i} $$

(20)

At the interface $z = 0$, the total tangential fields must be equal in both sides. For the electric fields, assuming that there are no fields traveling in the negative $z$ direction in the medium 3:

$$ E_3^* = E_{20}^* + E_{20}^+ $$

(21)

$\Gamma$ relates the fields $E_{20}^*, E_{20}^+$, therefore:

$$ E_3^* = E_{20}^* \left( 1 \right) $$

(22)

Substituting $E_{20}^*$, from (22), and $E_3^+$ by $S_{21}E_i^*$ in (20), into (15) and (16) and replacing the ratio between the characteristic impedances by the reflection coefficient $\Gamma$:

$$ E_i^* + E_i = \frac{S_{11}E_i^*}{\left( 1 \right)} \left( T^1 + T \right) $$

(23)

$$ E_i^* - E_i = \frac{1}{\left( 1 + S_{21}E_i^* \right)} \left( T^1 + T \right) $$

(24)

Adding (23) and (24), eliminating $E_i^* - E_i$ it is possible to isolate the transmission coefficient through the slab:

$$ S_{21} = \frac{T \left( \frac{1}{2} \right)}{1 - \frac{1}{\tau^2}} $$

(25)
Given that the dielectric slab is symmetrical and the material is isotropic and homogeneous, the scattering parameters matrix is completely specified by making $S_{22} = S_{11}$ and $S_{12} = S_{21}$.

IV. SIMPLE MODEL FOR NON-RESONANT METHODS

A. NRW algorithm

Consider a sample, as shown in Fig. 3, inside a coaxial cable with a termination impedance connected immediately after the sample ($d_L = 0$) or the medium 3 with an intrinsic impedance different of that of the medium 1 in free-space. In both cases, it is possible to model the system as a slab represented by its scattering parameters and loaded by impedance $Z_L$ or an infinite medium of intrinsic impedance $\eta_L$.

![Fig. 3. MUT in transmission line and free space with load.](image)

The reflection coefficient at the input can be obtained from the scattering parameters and the reflection coefficient at the load from [19] [20]:

$$G_{in} = S_{11} - \Delta_S G_L$$

(26)

Where $\Delta_S = S_{11}S_{22} - S_{12}S_{21}$. Considering the sample reciprocity:

$$G_{in} = \frac{S_{11}}{1} \left( \frac{S_{11}^c}{S_{11}} \right) L$$

(27)

If the load impedance is made equal to the characteristic impedance of the line loaded with the sample, the reflection coefficient at the input will be that of an infinite sample. This is due to the fact that, without reflection at the second interface, the wave will only exist in the positive direction from the input of the sample. Any load or infinite slab with the same impedance as the medium being measured will present the same reflection coefficient $\Gamma$ when considered in relation to the input medium. From these considerations, it follows that if the substitution $G_{in} = G_L = \Gamma$ is done in (27), it is possible to obtain the reflection coefficient at the interface as a function of the slab scattering parameters:

$$\left(1 \quad S_{11} \right) = S_{11} \left( S_{11}^c \quad S_{21}^c \right)^2 \frac{\left(S_{11}^c + 1 \right)}{S_{11}^c} + 1 = 0$$
\[ G = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}} \pm \sqrt{\left(\frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}\right)^2 - 4S_{11}^2} \]  

The sign in (28) must be chosen in a way that \(|\Gamma| \leq 1\). Defining:

\[ K = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}} \]

Equation (28) can then be written as:

\[ G = K \pm \sqrt{K^2 - 1} \]

Equations (29) and (30) are presented in [6] and [7] as a fundamental part of the NRW algorithm.

The scattering matrix relates the electric fields in the sample, as shown in Fig. 1, in the form:

\[ E_1 = S_{11} E_1 + S_{12} E_3 \]
\[ E_3^+ = S_{21} E_1^+ + S_{22} E_3 \]

The incident field in the load is \(E_3^+\) and the reflected is \(E_3^\). The reflection coefficient at the load, \(\Gamma_L\) is given by:

\[ \Gamma_L = \frac{E_3^-}{E_3^+} \]

Since the system is symmetrical \(S_{22} = S_{11}\) and the material is isotropic and homogeneous, then \(S_{12} = S_{21}\). From (31), \(E_i = \Gamma_L E_i^+\), the above equation system can be written as:

\[ E_1 = S_{11} E_1^+ + S_{21} \Gamma_L E_3^+ \]
\[ E_3^+ = S_{21} E_1^+ + S_{11} \Gamma_L E_3^+ \]

We can add these two equations and obtain an equivalent system with the same solution. The sum result is that:

\[ E_3^+ + E_1 = \left(S_{11} + S_{21} \Gamma_L\right) E_1^+ + \left(S_{21} + S_{11} \Gamma_L\right) E_3^+ \]

Since in the proposed situation, there is not a reflected wave inside the sample and \(\Gamma_m = \Gamma_L = \Gamma\), it follows:

\[ E_3^+ + E_1 = \left(S_{11} + S_{21} \Gamma\right) E_1^+ + \left(S_{21} + S_{11} \Gamma\right) E_3^+ \]

Similarly, the propagation factor is given by:

\[ T = \frac{E_3^+}{E_1^+} \]

From these equations we can evaluate expressions for \(E_3^+ \) e \(E_1^+\), which are substituted in (32), with \(\Gamma_L = \Gamma\):

\[ TE_1^+ + E_1^+ = \left(S_{11} + S_{21} \Gamma\right) E_1^+ + \left(S_{21} + S_{11} \Gamma\right) TE_1^+ \]
then:

$$T = \frac{S_{11} + S_{21}}{1 + (S_{11} + S_{21})}$$

(35)

Equation (35) shows the propagation factor as a function of the scattering parameters and the reflection coefficient at the interface presented in [5] and [6]. The NRW algorithm to determine the permittivity from the MUT scattering parameters is fulfilled when (2) is considered and then:

$$\varepsilon_r = \left(\frac{1}{1+\Gamma}\right)^2$$

(36)

And from (4):

$$\varepsilon_r = \left(\frac{j\epsilon}{d \ln(T)}\right)^2$$

(37)

The equations (36) and (37), isolated or combined, can be used for permittivity determination [21]. The use of (36), as described in [12] will result in a permittivity explicit expression, independent of the sample size. However this leads to indeterminations when the sample length is a multiple of half wavelength in low loss materials. The authors conducted an uncertainty analysis as a function of the permittivity of the measured material and of the sample size. Equation (37) does not show these problems, but it depends on the sample length, which leads to phase ambiguity problems since T is complex and its logarithm may have infinite solutions [22].

B. Reflection only methods

If in fig. 3, since \(d_L = 0\) and the load is a short-circuit, we have the method known as SCTL (short-circuit transmission line). This method is also applied to the free-space [4] where the short-circuit is made through a metal back (metal-back method). Other load types are possible. The model in fig. 3 can be used with any load. The sample scattering parameters, as functions of the propagation factor \(T\) and of the reflection coefficient at the interface \(\Gamma\), are given in (19) and (25). In a distinct approach from the deduction of the NRW algorithm, which is intended to write \(\Gamma\) as a function of scattering parameters only, we now want an expression for the input reflection coefficient \(\Gamma_{in}\), given by (27), as a function of the factor \(T\) and of the coefficient at the interface \(\Gamma\). When substituting (19) and (25) in (27) (obtained from (26)), then:

$$\Gamma_{in} = \frac{1}{2} \left(1 + \frac{T^2}{2\sqrt{\epsilon_r}}\right) \left(\frac{\epsilon_r}{\epsilon_{ref}}\right)$$

(38)

C. Double reflection methods – same size samples and different loads.

It is possible to obtain an explicit equation for the permittivity from (38) through the double reflection method [11][15] (also known as double impedance method [14]) or when considering the
same sample with two different loads. These measurements result in the input reflection coefficients $\Gamma_1$ and $\Gamma_2$ from the respective loads $\Gamma_{L1}$ and $\Gamma_{L2}$. For each one of the loads the propagation factor $T$ can be isolated in (38) with $\Gamma$ given by (2):

$$T^2 = \left[ \frac{L}{\sqrt{r}} \left( \sqrt{r} + 1 \right) + \frac{1}{\sqrt{r}} \right] \frac{\ln \left( \sqrt{r} + 1 \right) + \sqrt{r} + 1}{\ln \left( \sqrt{r} - 1 \right) + \sqrt{r} - 1}$$

(39)

Thus, if $\Gamma_{L1} = -1$ (short circuit) in the first measurement and $\Gamma_{L2} = 1$ (open circuit) in other measurement, are applied to equation (39) and compared, the permittivity as a function of two reflection coefficients $\Gamma_1$ and $\Gamma_2$ is:

$$e_r = \frac{Y_a}{Y_c}$$

(40)

The normalized input admittance of a transmission line is given by [19]:

$$y = \frac{Y}{Y_0} = \frac{1}{1 + 1}$$

(41)

Therefore, the permittivity of a sample, when obtained from two measurements, one terminated in a short-circuit and the other terminated in an open-circuit, is given by:

$$r = Y_a Y_c$$

(42)

The permittivity as a product of a short-terminated line admittance ($y_a$) by an open-circuit terminated line admittance ($y_c$) has already been shown in [14].

D. New double reflection explicit equations

When measuring the reflection coefficient of a short terminated sample and, then, of an impedance matched terminated sample ($\Gamma_{L1} = -1$ e $\Gamma_{L2} = 0$), it is possible to obtain another explicit equation from (39):

$$r = \frac{2}{2 - 1} \frac{3 + 1}{2 + 1}$$

(43)

Applying the same procedure, but with an open-circuited load in place of the short-circuited one and, then, of an impedance matched terminated sample ($\Gamma_{L1} = 1$ e $\Gamma_{L2} = 0$), the permittivity is now given by:

$$r = \frac{2}{2 - 1} \frac{3 + 1}{2 + 1} \frac{1}{1}$$

(44)

This equation has been derived earlier [15], but its derivation uses a different procedure.

The equations (43) and (44) are particular cases of a general equation. Given any two loads $\Gamma_{L1}$ and $\Gamma_{L2}$ (with two measurements $\Gamma_1$ and $\Gamma_2$ being done with these two loads), the general explicit equation for permittivity is:
E. Double reflection methods – same loads and different size samples

Using two samples with different lengths, arranged on a short-circuited line, as shown in Figure 4, it is possible to obtain the permittivity and the permeability. This procedure appears in [11] and [23].

\[ r = \frac{\Gamma_1 + \Gamma_2}{\Gamma_1 - \Gamma_2} = \frac{L_1 + L_2}{L_1 - L_2} \] (45)

\[ \Gamma_1 = -1 \quad \Gamma_L = -1 \]

Fig. 4. Short-circuited lines with different sizes samples.

An explicit equation for the permittivity can be obtained considering \( \Gamma_L = -1 \) in (39):

\[ T^2 = \left[ \frac{1 + \sqrt{r} + 1}{1 + \sqrt{r}} \right]^2 \] (46)

By measuring with sample widths \( d_2 = \alpha \, d_1 \) the squared propagation factor is given by:

\[ T^2 = \left( \frac{\left[ 2 + \sqrt{r} + 1 \right]}{\left[ 2 + \sqrt{r} + 1 \right]} \right)^{\frac{1}{\alpha}} \] (47)

where \( \alpha \) is a scaling factor.

Knowing the relationship between the widths and measuring the reflection coefficients, it is possible, for certain \( \alpha \) values, to obtain explicit expressions for the permittivity considering the equation:

\[ \left[ \frac{1 + \sqrt{r} + 1}{1 + \sqrt{r}} \right]^2 = \left[ \frac{2 + \sqrt{r} + 1}{2 + \sqrt{r} + 1} \right]^2 \] (48)

In [11], the widths are set as \( d_2 = 2 \, d_1 \) (or \( \alpha = 2 \)). Equation (48) can then be solved explicitly, obtaining the permittivity:

\[ r = \left( \frac{1}{2} + 1 \right) \left( \frac{3}{2} + 1 \right) \] (49)

Employing the same procedure, starting from equation 39 but forcing the samples to end with a matched load or an absorbing material in free space, another explicit equation for the permittivity can be obtained:
\[ r = \left( \frac{1}{1 + 1} \right) \left( \frac{1}{2} + \frac{1}{2} \right) \] (50)

V. Accuracy

To estimate the uncertainty of the new equations, the Monte Carlo method is applied. The error sources considered are the finite accuracies of the measured reflection coefficient (within 3% of the nominal value for amplitude and phase) and of the load impedance (taken to be within 1% of nominal value). The combined effect of these error sources is computed for a population of 5000 samples in a rectangular distribution. A low-loss material with \( \varepsilon = 4 - 0.2j \) and \( 25 \text{ mm width} \) was used.

The standard deviation in permittivity generated by these error sources when applied to equations (36) (obtained from (28), NRW method), (40) and (43) are shown in Fig. 5 and Fig. 6, for the real and imaginary parts of the permittivity, respectively. When the same errors sources are applied to (49) and (50), the results are shown in Fig. 7 and Fig. 8, for the real and the imaginary parts of the permittivity, respectively.

Fig. 5. Same size – different loads. Standard deviation of the real part of the permittivity.

Fig. 6. Same size – different loads. Standard deviation of the imaginary part of the permittivity.

In the Fig. 5 and Fig. 6 it can be observed that the new explicit equation, (43), has smaller uncertainty in the frequencies which are multiple of half-wavelengths (3, 6 and 9 GHz) in comparison...
to the traditional method of (40). Minimal uncertainty in frequencies 1.5, 4.5 and 7.5 GHz is found for (43), whereas (40) has an instability. It also can be noted that (43) has, in the entire band, a lower uncertainty for the imaginary part (when compared to the NRW method). The uncertainty for the real part of permittivity in quarter-wavelength frequencies (1.5, 4.5 and 7.5 GHz) is slightly higher for (43) than for the NRW method, but for half-wavelength frequencies the precision of (43) is higher than the NRW.

Fig. 7. Same load – different sizes. Standard deviation of the real part of the permittivity.

Fig. 8. Same load – different sizes. Standard deviation of the imaginary part of the permittivity.

In Fig. 7 and Fig. 8, it can be observed, that the different-size-samples method, which uses matched loads, shows a lower error along most part of the band.

For both equations the uncertainties get smaller (and closer to one another) as the frequency increases. This can be due to the larger number of wavelengths inside the material sample width (a virtual thickening), resulting in larger attenuation and less signal being reflected at the termination.

VI. CONCLUSION

A new general model for the non-resonant permittivity measurement method was presented. From
this model the equations for classical NRW algorithm and SCTL method can be derived. Explicit equations to determine the permittivity were obtained from the new model. In addition to this, two new equations for the double reflection method were evaluated. One of them uses a short circuit load and a matched load and the other uses an open-circuit load and a matched load. A new equation is also obtained for the method with different sizes terminated in the same load, in this case, a matched one. The uncertainty of the new equations is calculated using the Monte Carlo method and, in both cases, it is lower than the classical methods. These methods were used for TEM waves in the free-space and in transmission lines. However, they could be easily extended for waves and samples in rectangular waveguides.

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