RESPONSE OF MICROSTRIP-RING RESONATORS DUE TO PICO SECOND TIME DEPENDENT PULSE

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Abstract

The response of microstrip ring resonator due to time dependent picosecond pulse has been analyzed using time domain technique. Equi-impedance 50 ohms microstrip ring resonators with various mean radii have been designed and fabricated on glass-epoxy type PCB board material. The responses were transformed in the frequency range 1-18 GHz using the Fourier transformation. Further the resonance frequencies in fundamental and higher order modes were determined. These results are in agreement with corresponding results determined by frequency domain transmission technique using microwave scalar network analyzer. These values were used to study the dispersion in microstriplines. The experimental results are in good agreement with the theoretical results due to the Edwards and Kirschning-Jansen formulae.

Index Terms -- Microstripline, characteristic impedance, microstrip ring resonator (MRR), time domain, mV- pulse, rise time, Fourier transform, resonance frequency, effective dielectric constant, dispersion.

I - INTRODUCTION

Microstrip ring resonators, MRRs, are frequency selective planar microwave devices, which are potentially used to study microstripline discontinuities, microstrip dispersion, phase velocity and dielectric behaviour in frequency swept domain techniques [1]-[3]. Time domain spectroscopy (TDS) in the slow and fast transient mode has been extensively used to determine electrical dielectric properties of various polar liquids [4], [5]. The time domain techniques in reflection and transmission modes could be employed to test the quality of microwave cables and connectors regarding cable losses. Time domain studies can also be made to evaluate cross talk in multi speed digital transmission lines [6]. In the present paper picosecond time domain transmission mode has been used to evaluate equi-impedance microstrip ring resonators response, later which Fourier transformed into frequency domain yielding resonance frequencies corresponding to fundamental as well as higher modes. Further these resonance frequencies are used to evaluate frequency dependent effective dielectric constant over the frequency range 1-18 GHz. The details regarding fabrication of MRRs is given in section II. The time domain measurements are described in section III. A brief relevant theoretical description of MRR is accounted in section IV. The results
and discussions are given in section V. Finally, the work is concluded in the last section VI.

II - EXPERIMENTAL

The Microstrip ring resonators have been designed and fabricated on fire resistant glass-epoxy (FR-4) PCB board material by standard UV chemical photo etching printed circuit board (PCB) technology. The electrical and structural parameters of glass-epoxy (FR-4) board material are dielectric constant ($\varepsilon_r = 4.6$), dielectric height ($h = 1.6$ mm) and copper weight or thickness ($t = 70$ µm). The accuracy claimed for UV chemical-photo etching is of the order of 0.02 mm by the three-layer dry film lamination method [7].

In the design of a microstrip line, the characteristic impedance $Z_0 = 50$ Ohms is fixed, to keep the reflections as small as possible, and the corresponding critical line width $w$ to be obtained from CAD design curves [8]-[10]. Knowing the electrical and structural parameters for PCB board material (FR-4), the critical line width $w = 2.96$ mm results in the characteristic impedance $Z_0 = 50$ Ohms. The design is based on Hammerstadt’s general-purpose synthesis and analysis formulae [11]. The mean radius of a ring $R_m$ is varied between 6 mm to 9.5 mm in the step of 0.5 mm. Further a pair of symmetrical feed lines are diametrically placed about the ring with coupling gap ($s = 0.2$ mm) so as to obtain sufficient power transmission through the ring. The edges of feed lines facing towards the ring are made circular such that the capacitive coupling gap is uniform. The precision 3.5-mm co-axial to microstripline SMA connectors are properly soldered to provide good electrical coupling and a mechanical rugged support.

III - EXPERIMENTAL SET UP

The digitizing sampling oscilloscope HP-54754A along with HP-54754A TDR/TDT plug in module has been used to investigate the time domain response from MRRs in time domain transmission (TDT) mode [12], [13]. The channels 3 and 4 are selected as source and destination channels as shown in fig. 1. A fast rising voltage step with height of 200 mV and rise time about 40 ps from channel 3 is driven through 50 Ohms coaxial cable of length 1 m which is connected to input feed line of microstrip ring resonator. The other feed line is connected to the destination channel 4 via SMA connector, as shown in fig. 1. The time window of 10 ns is found optimum to observe the complete waveform response from MRR in transmission mode, which is monitored and digitized into 1024 points by the sampling oscilloscope. Each point is averaged over 64 counts to improve the signal quality. The stored digitized data is transferred to PC through standard 1.44-MB diskette drive for further analysis. It is observed that, the MRR response in TDT mode is oscillatory with decreasing amplitude. The stored time domain data is analyzed into frequency domain by using Fast Fourier Transform (FFT) facility available in the H.P’s digitizing oscilloscope. The rectangular and Hanning time windows were used to digitize the transient time domain signal obtained by the transmitted voltage step through a microstrip ring resonator. The frequency resolution is obtained in Hanning time window thereby FFT yielding resonant frequencies in fundamental and higher order modes (harmonics).

To confirm the resonant frequencies of MRRs as obtained by FFT, in frequency domain Hewlett Packard’s scalar Network analyzer HP8757C along with HP8350B oscillator, HP83592B RF plug-in module (0.01-20 GHz), power splitter 11667A (DC-18 GHz) and a detector 85025A (.01-18 GHz) were used in the transmission mode. The resonant frequencies of MRRs in various
modes in time and frequency domain were found same within the experimental accuracy. These resonant frequency values are used to obtain frequency dependent effective dielectric constant up to 18 GHz.

Fig. 1. A typical microstrip ring resonator connected to the sampling oscilloscope in transmission mode with 200 mV step height and 40 ps rise time.

IV - THEORETICAL BACKGROUND

Hammerstadt’s general-purpose formulae for synthesis and analysis are used to compute characteristic impedance ($Z_0$) of a microstrip feed line. The microstrip ring resonator has been generally studied by (i) the transmission line approach and (ii) the wave guide approach [14]. The prototype microstrip ring resonator can be considered to be a cylindrical cavity with electrical walls on the top and bottom and the magnetic walls at the side border [15]. In such a model the E. M. fields are assumed to be confined to the dielectric volume between the perfectly conducting ($\sigma=\infty$) ring conductor and the ground plane and the magnetic walls with ($\mu=\infty$) having cylindrical shape at the inner and outer radii of the ring. Considering Maxwell’s equations for the E.M. field components in the ($r, z, \Phi$) coordinate system we have,

$$
\begin{align*}
E_z &= \{A J_n (kr) + B N_n (kr)\} \cos(n\Phi) \\
H_r &= \left(\frac{n}{j\omega \mu_0 r}\right) \{A J_n (kr) + B N_n (kr)\} \sin(n\Phi) \\
H_\Phi &= \left(\frac{k}{j\omega \mu_0}\right) \{A J'_n (kr) + B N'_n (kr)\} \cos(n\Phi)
\end{align*}
$$

where $J_n$ and $N_n$ are the Bessel’s functions of first and second kind of order $n$ and $J'_n$, $N'_n$ are
derivatives of Bessel’s functions w. r. t. kr, \( k = \sqrt{\left(\varepsilon_0 \varepsilon_{\text{eff}} \mu_0\right)} \) is the wave number and A, B are arbitrary constants to be evaluated by using the boundary conditions.

Similarly the other solution can also be obtained as,

\[
E_z = \begin{pmatrix}
AJ_n(\nu) + BN_n(\nu) \sin(n\Phi) \\
\frac{n}{j\omega \mu_0} \begin{pmatrix}
AJ_n(\nu) + BN_n(\nu) \cos(n\Phi) \\
AJ_n'(\nu) + BN_n'(\nu) \sin(n\Phi)
\end{pmatrix}
\end{pmatrix}
\]

The solutions (1) and (2) can be interpreted corresponding to even and odd modes respectively.

Application of the magnetic wall boundary condition \( \frac{\partial E_z}{\partial r} = 0 \) at \( r = R_i \) and \( R_o \) (i. e. at inner and outer radius), assumed for wide strips leads to the following equations for resonant modes,

\[
\frac{A}{B} = -\frac{N_n'(kR_i)}{J_n'(kR_i)} \quad (3)
\]

\[
\frac{A}{B} = -\frac{N_n'(kR_0)}{J_n'(kR_0)} \quad (4)
\]

\[
J_n'(kR_i)N_n'(kR_0) = J_n'(kR_0)N_n'(kR_i) \quad (5)
\]

and

\[
k = \frac{\omega}{c} \sqrt{\varepsilon_{\text{eff}}} \quad , \quad (6)
\]

where \( \omega \) is the resonance frequency in radians per second, \( c \) is the speed of light in free space, \( \varepsilon_{\text{eff}} \) is the effective dielectric constant and \( n \) is the integer, known as azimuthal mode number [16].

For narrow strips \( (R_i = R_o) \) equation (5) reduces to,

\[
\left[ k_{R_i}^2 - n^2 \right] J_{n-1} - k_{R_i} N_{n-1} - k_{R_i} N_{n-1} J_{n-1} = 0 \quad (7)
\]

The second factor is nonzero therefore we can write,

\[
k_{R_m} = n \quad (8)
\]
where $k$ is the wave number and can be expressed by

$$k = \frac{2\pi}{\lambda_g},$$

(9)

which on substitution in equation (8) gives,

$$1 = 2\pi R_m = n\lambda_g$$

(10)

Thus this is the essential condition of resonance, i.e.; the length of the ring must be an
integral multiple of guide wavelength. As $n\lambda_g = \frac{c}{\sqrt{\varepsilon_{\text{eff}}}}$, we obtain the formula for resonance
frequency for the ring resonator as,

$$f_r = \frac{nc}{2\pi R_m \sqrt{\varepsilon_{\text{eff}}}}$$

(11)

The above formula has been used in the present experimental work for computation of
resonance frequencies. For dispersion, the frequency dependent effective dielectric constant $\varepsilon_{\text{eff}}(f)$
may be obtained as,

$$\varepsilon_{\text{eff}}(f) = \frac{n^2 c^2}{4\pi^2 R_m^2 f_r^2}$$

(12)

It has been reported that for high impedance ring resonators having characteristic impedance
110 Ohms, the dispersion of the effective dielectric constant is small about 1.5% whereas for
smaller impedance rings (20 Ohms) it is large about 2.5% in the frequency range 2-8 GHz.[2].

For alumina type substrates Getsinger suggested following formulae [14],

$$\varepsilon_{\text{eff}}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{\text{eff}}}{1 + G \left(\frac{f}{f_p}\right)^2}$$

(13)

where

$$f_p = \frac{Z_0}{2\mu_0 h}$$

(14)

$$G = 0.6 + 0.009Z_0$$

(15)

For alumina and sapphire type substrates Edwards and Owens suggested an accurate
expression for dispersion as [15],
\[ e_{\text{eff}}(f) = e_r - \frac{e_r - e_{\text{eff}}}{1 + \left(\frac{f}{Z_0}\right)^{1.33} \left(\frac{0.43f^2}{f^2 - 0.09f^3}\right)} \]  

(16)

where \( h \) is in mm and \( f \) is in gigahertz.

Kirschning and Jansen’s basic expression for dispersion with \( f \) in GHz and thickness \( h \) in cm is given as, [17]

\[ e_{\text{eff}}(f) = e_r - \frac{e_r - e_{\text{eff}}}{1 + P(f)} , \]  

(17)

And the form of frequency function is

\[ P(f) = P_1 P_2 \left[ (0.1844 + P_3 P_4) f h \right]^{0.5763} \]  

(18)

where

\[
\begin{aligned}
P_1 &= P' - P'' , \\
P' &= 0.27488 + \left[0.6315 + 0.525/(1 + 0.157f)^{20}\right] h , \\
P'' &= 0.065683 \exp(-8.751w/h) , \\
P_2 &= 0.33622 \left[ 1 - \exp(-0.03442e_r) \right] , \\
P_3 &= 0.0363 \exp(-4.6w/h) \exp\left(-\left(\frac{fh}{3.87}\right)^{4.97}\right) , \\
P_4 &= 1 + 2.75 \left[ 1 - \exp\left(-e_r/15.916\right) \right] \left[ \left(\frac{f}{\lambda_0}\right)^2 + 1\right] .
\end{aligned}
\]  

(19)

An accuracy of better than 0.6 per cent is claimed for all frequencies up to 60 GHz. The restrictions applying are very wide namely,

\[
\begin{aligned}
1 &\leq e_r \leq 20 , \\
0.1 &\leq w/h \leq 100 , \\
0 &\leq h/\lambda_0 \leq 0.13.
\end{aligned}
\]  

(20)

The time domain transmission (TDT) response of microstrip ring resonator (MRR) is analyzed into frequency domain by considering Fourier transform, given as [18], [19]

\[ \Delta \omega = \int f(t)e^{-\sqrt{\omega/\lambda_0}}dt \]  

(21)

A computer program is developed for Fourier transform by using summation method. It is also modified for data smoothening to reduce the noise error present in the base line in the frequency range 1-18 GHz. The frequency step of 0.02 GHz is used and the corresponding frequency data is in good agreement with as obtained by rectangular or Hanning type time window based in built FFT analysis.
V - RESULTS AND DISCUSSION

As discussed in experimental set up, the response of a microstrip ring resonator is observed on digitizing sampling oscilloscope using a 200 mV step is transmitted through the ring, which is capacitively coupled to the feed lines. After proper magnification of transmitted voltage, the step response so obtained is oscillatory with decreasing amplitude or as damped oscillations. This response must be observed in an optimum time window, as small time window of the order of 1ns, the device response waveform may be clipped off resulting in a loss of information while wide time window greater than 20 ns, may give unwanted noise signals. The decrease in amplitude in time domain may suggest the lossy nature of microstriplines or it may be possibly due to the presence of various frequency components with decreasing amplitudes successively in a 200 mV step of 40 ps rise time. A typical microstrip ring resonator response waveform in time domain transmission mode is shown in fig. 2. The Fast Fourier Transform (FFT) of time domain response waveform in rectangular or Hanning time window gives the required frequency domain data analysis. The Hanning window provides frequency resolution whereas rectangular window is useful for transient signals. The typical MRR having mean radius of 9.5 mm provides the corresponding Fats for rectangular and Hanning time windows as shown in Fig. 3 and 5. Further data smoothening can be made to reduce the base line error in the Fourier Transform as shown in fig. 6.

The experimental technique in frequency domain as explained earlier, provides identical frequency measurements as expected, is shown for the same sample (Rm=9.5 mm) in fig. 4. The experimental values regarding the resonance frequencies in fundamental and higher order harmonics in time and frequency domain are reported in table 1. These experimental results are plotted along with the theoretically generated curves for n=1,2,3,4 against the normalized mean size w/Rm for equi-impedance rings. Theoretically resonance values are computed by considering the effective dielectric constant as 3.46, for glass-epoxy (ε_r=4.60) board material, which is obtained by the Owens formula which holds for alumina or sapphire type substrates. It is noticed from the fig. 7 that time and frequency domain experimental values for resonance frequencies coincide very well but lower than theoretically computed values for higher order resonance, this may be due to frequency dependent nature of effective dielectric constant, i.e. dispersive nature of microstriplines.

To verify the dispersive nature of microstriplines, the corresponding theoretical curves using the Edwards and Kirschning-Jansen formulae are used as it covers wide range of dielectric constants for comparison with the experimental values. Fig. 8 shows the plot for experimental values and theoretically fitted curves for \( \varepsilon_{\text{Eff}}(f) \) against frequency, which agree well within 1 % in \( \varepsilon_{\text{Eff}}(f) \). It is also observed that for large ring size the Kirschning-Jansen type dispersion formulae are applicable while for smaller rings Edwards type dispersion fits nicely up to 18 GHz. The static or low frequency effective dielectric constant for Edwards type is 3.30-3.35 whereas 3.05 for Kirschning-Jansen type dispersion fits well, which are slightly less than 3.46 as obtained by Owens formula.
Fig. 2. A typical time domain response waveform on sampling oscilloscope from a microstrip ring resonator having mean radius $R_m=9.8$ mm in transmission mode.

Fig. 3. An FFT of MRR ($R_m=9.8$ mm) response waveform obtained from sampling oscilloscope using a rectangular window in time domain.
Fig. 4. A typical microstrip ring resonator response in transmission mode on scalar network analyzer in frequency domain having mean radius $R_m = 9.8$ mm.

Fig. 5. An FFT of MRR ($R_m = 9.8$ mm) response waveform obtained from sampling oscilloscope using a Hanning window in time domain.
Fig. 6. The Fourier Transform obtained from MRR ($R_m=9.8$ mm) response waveform in a time window of 10 ns by using curve smoothening technique.

Fig. 7. Equi-impedance MRR Resonance frequencies in fundamental and higher order harmonics obtained in time and frequency domain for normalized mean ring size $w/R_m$. Continuous lines are drawn as theoretical resonance frequencies computed by using $\varepsilon_{\text{Eff}}=3.46$ (Owens formula). Legends ‘•’ and ‘▲’ are used for time domain and frequency domain experimental resonance frequencies. The integer $n=1,2,3,4$ gives the order of resonance frequencies.
Fig. 8. Dispersion curves drawn for MRRs fabricated on glass-epoxy type (FR-4) substrate board material, indicating error bars on experimental resonance frequencies with an accuracy of 0.5% in $\varepsilon_{\text{Eff}}(f)$. Legends ‘•’, ‘▲’ and ‘■’ are used for rings having mean radii 9.5, 9.0 and 8.5 mm respectively. The dotted curved line is the Kirschning-Jansen type dispersion formulae fitted for $\varepsilon_{\text{Eff}} = 3.05$. The continuous lines are drawn by using Edwards type dispersion formulae fitted for $\varepsilon_{\text{Eff}} = 3.30$ and $\varepsilon_{\text{Eff}} = 3.35$. Figure shows the dispersion dependence nature on the ring size.

Table 1: Experimental resonance frequencies of microstrip ring resonators in fundamental and its harmonics i.e. higher order resonance modes for various mean radii in time and frequency domains.

<table>
<thead>
<tr>
<th>Mean radius of microstrip ring $R_m$ in mm</th>
<th>Time domain results of MRR resonance frequencies in GHz</th>
<th>Frequency domain results of MRR resonance frequencies in GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1$  $f_2$  $f_3$  $f_4$</td>
<td>$f_1$  $f_2$  $f_3$  $f_4$</td>
</tr>
<tr>
<td>6.0</td>
<td>4.40  8.28  12.06  -</td>
<td>4.355  8.28     -  -</td>
</tr>
<tr>
<td>6.5</td>
<td>4.02  7.74  11.28  -</td>
<td>4.053  7.710   11.257  -</td>
</tr>
<tr>
<td>7.0</td>
<td>3.78  7.27  10.66  13.76</td>
<td>3.777  7.215   10.597  -</td>
</tr>
<tr>
<td>7.5</td>
<td>3.48  6.80  9.92   12.98</td>
<td>3.5025  6.775  9.938  -</td>
</tr>
<tr>
<td>8.5</td>
<td>3.04  6.02  8.88   11.64</td>
<td>3.100  6.040  8.890  11.620</td>
</tr>
<tr>
<td>9.0</td>
<td>2.90  5.70  8.44   11.04</td>
<td>2.920  5.710  8.410  11.020</td>
</tr>
<tr>
<td>9.5</td>
<td>2.80  5.48  8.10   10.56</td>
<td>2.800  5.470  8.080  10.570</td>
</tr>
</tbody>
</table>
VI - CONCLUSIONS

The response of microstrip ring resonator due to 200 mV step pulse in transmission mode is presented and further used for Fourier transform analysis in frequency domain. The resonance frequencies so obtained are verified in frequency domain by using microwave scalar network analyzer. The results are well in agreement within the experimental uncertainties, of 1% as expected. The dispersion in microstriplines up to 18 GHz is confirmed by using the Edwards and Kirschning-Jansen type formulae. The curve fittings for effective dielectric constant indicate slightly lower values than as computed by Owens formula, possibly due to the epoxy type material present in the board material.

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