EHBSim: MATLAB-BASED NONLINEAR CIRCUIT SIMULATION PROGRAM
(HARMONIC BALANCE AND NONLINEAR ENVELOPE METHODS)

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Abstract
This paper presents a nonlinear circuit simulation program called EHBSim implemented in Matlab. This program offers as simulation methods the Harmonic Balance method, and especially the Nonlinear Envelope method. Equations systems that allow the simulation of any circuit architecture were also used. The program has original algorithms implemented to solve the associated circuit equation. The description of the Nonlinear Envelope Method as well as a performance comparison between those two simulation methods, through the analysis of multitone excited circuits, are also carried out.

I - INTRODUCTION
In the simulation of nonlinear electric circuits excited by modulated carriers, there are two options for representing the modulation/demodulation process: passband and baseband. In passband simulation, the carrier signal is included in the model of the electric elements (linear and nonlinear elements). The frequency of the carrier signals is usually much higher than the highest frequency of the input message (baseband) signal. According to the Nyquist sampling theorem, the sampling frequency of the simulation must be at least twice as high as the carrier frequency in order to allow the recovery of the message. However, the simulation of a high frequency signal is very slow and consequently inefficient when traditional time domain integration methods [1,2] are used. To speed up the simulation process, a baseband simulation can be used. A baseband circuit simulation method, known as the Nonlinear Envelope Method (NLEM) [1,3], which is a low-pass method, handles the complex envelopes of the modulated signals. The following sections present the mathematical definition of a complex envelope, as well as the models for basic linear circuit elements and the way to characterize nonlinear elements in the envelope domain.

A Matlab-based nonlinear electric circuit simulation program called EHBSim (Envelope & Harmonic Balance Simulator) is also presented in this paper. The EHBSim program presents as main simulation method the Nonlinear Envelope Method. This method works in both time and frequency domains, and allows the analysis of circuits excited by modulated carriers by extracting the complex envelopes of the signals, which can be narrow or wideband, analog or digital. Since
this technique is limited to the analysis of complex envelopes, it demands reasonably low computational costs. Thus it allows efficient analysis of RF telecommunication systems excited by arbitrary modulated carriers. EHBSim also implements the Harmonic Balance Method (HBM) \cite{1,4,5}. Another feature of EHBSim is that the implemented equation system yields the simulation of any circuit topology. EHBSim uses the Newton-Raphson algorithm \cite{1,2,4} associated with continuation techniques \cite{1,2} as iterative method for solving the circuit equations.

II - EHBSim SIMULATION METHODS

II.1 - Harmonic Balance Method

The several simulation techniques currently in use can be classified as time-domain, frequency-domain, and time and frequency-domain techniques. Among these techniques, the well-established Harmonic Balance Method has shown greater efficiency through the last decade. However, this technique presents serious difficulties when the circuit is excited by modulated signals, demanding, in these cases, high computational costs. EHBSim implements this method. It works in both time and frequency domains. In this method the circuit equation is formulated in the frequency domain, but, because of the impossibility to obtain the models of the nonlinear elements in the frequency domain, the evaluation of the nonlinear functions (current, voltage and charge) are performed in the time domain. To perform this evaluation, we first take the inverse Fourier transform of the command (current or voltage) of the nonlinear function, operate the nonlinear function, and then use the forward Fourier transform to convert back this function to the frequency domain. Since the system of equations is represented in the frequency domain, using Fourier coefficients, it cannot give the transient solution of the circuit. Therefore, the HBM is limited to the search of the steady-state solution.

The first step towards the application of the HBM is to build the circuit equation. It is represented in the frequency domain and is a vector composed by the combination of the element models (obeying the element laws) and the topology of the circuit (obeying Kirchhoff’s laws). This equation contains information about the models of linear elements (resistors, capacitors, inductors, transmission lines, etc.), nonlinear elements (voltage, current and capacitance), and sources (voltage and current). The equation system takes the form:

\[
F(C_{NL}(\Omega) - A_1G_L(\Omega) - A_2G_{NL}(C_{NL}(\Omega)) - A_3C_{NL}(\Omega) = 0
\] (1)

Eq. (1) represents the segmented modified nodal equation \cite{1,4}. In this equation, only the minimal set of unknowns of the system are considered (commands of the nonlinear functions). In expression (1), \(\Omega\) is a vector composed of generic angular frequencies, \(C_{NL}(\Omega)\), \(G_{NL}(C_{NL}(\Omega))\) and \(G_L(\Omega)\) are, respectively, the Fourier coefficients of the nonlinear functions commands \(c_{NL}(t)\), the nonlinear elements characterized by nonlinear functions \(g_{NL}(c_{NL}(t))\), and the independent sources \(g_L(t)\). The matrices \(A_1\), \(A_2\) and \(A_3\) are composed by the models of the electric elements, obeying the circuit topology. They are also called transfer functions. Eq. (1) is generally solved using the Newton-Raphson algorithm associated with continuation techniques, based on the control of the excitation source levels. Thus, the level of the input sources of the circuit are reduced until convergence (solution) is achieved. Then the level is discretely increased, the solution found to a lower level is used as initial solution for a higher level, and the search of the
next solution starts. This process is repeated until the levels of the sources reach the desired value.

To simulate multitone electric circuits, one can use the multidimensional Fourier transforms, and more specifically, the multidimensional fast Fourier transforms, in order to take advantage of the gain in time and computational effort offered by them. In this case, \( \Omega \) represents a set composed by the fundamental frequencies of the circuit, their harmonics, and the summations and differences between frequencies of the fundamental components and their harmonics. The dimension of the multidimensional FFT [1,4] used is given by the number of fundamental frequencies of the electric signals.

The HBM is more accurate and efficient when the circuit is near linear and the electric signals are near sinusoidal. If the circuit to be simulated contains signals with abrupt transitions or if they are strongly nonlinear, such as those commonly present in telecommunication circuits, the HBM does not shows efficiency because: (1) many more frequencies are needed in order to represent correctly the signals of the electric circuit making the simulation of such circuit highly time-consuming; (2) the continuation techniques make more calls to the HBM as the nonlinearity increases. Even with the use of the FFTs to link the time and frequency domains and Krylov subspace methods [6] to solve the sparse linear equation (1), the use of the HBM to solve strongly nonlinear circuits presenting digital signals, which have abrupt transitions, is expensive and almost prohibitive.

II.2 - Nonlinear Envelope Method

The Nonlinear Envelope Method (NLEM) is capable of providing both transient and steady-state responses of an electric circuit. This method can be efficiently applied to the simulation of multi-excited modulated systems when the modulation signals are complex (e.g., QAM signals). This method takes into account only the slow variations of the modulated signals. In fact, the NLEM shows a much better performance than conventional transient analysis algorithms. In this method, one only needs to consider a number of sample points necessary to correctly represent the baseband components of the signals. Thus, the efficiency of this method, when it is compared with conventional transient methods, can be roughly estimated as the relationship between the number of sample points necessary to represent the modulated signal and the number of sample points necessary to represent the baseband (or modulating) signal. For example, a typical cellular telephone transmission has a 30 kHz modulation bandwidth riding on a 1 GHz carrier. Hence, an estimate for the gain achieved by using the NLEM instead of using conventional transient methods is over 3,000 in this case. Hence, if the envelopes (or complex modulating signals) change slowly if compared to the period of the carrier, the simulation by the NLEM will be very efficient.

Mathematical description of the complex envelope

A generic signal presenting two fundamental frequencies takes the form:

\[
    x(t) = \sum_{m=\infty}^{\infty} \sum_{n=\infty}^{\infty} X_{mn} e^{j(m\omega_1 + n\omega_2) t}
\]

(2)

where \( \omega_1 \) and \( \omega_2 \) are the fundamental angular frequencies and \( X_{mn} \) are the Fourier coefficients of \( x(t) \) that correspond to the frequencies \( m\omega_1 + n\omega_2 \). This description can be easily extended to
the case we have an arbitrary number of fundamental frequencies. Here only two fundamental
frequencies are presented for simplicity, but without loss of generality.

Expression (2) can be rearranged, resulting in:

$$x(t) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} X_{mn} e^{j\omega_0 t} e^{j\omega n t}$$  (3)

Equation (3) can be rewritten as:

$$x(t) = \sum_{n=-\infty}^{\infty} \overline{X}_n(t) e^{j\omega_0 t}$$  (4)

where

$$\overline{X}_n(t) = \sum_{m=-\infty}^{\infty} X_{mn} e^{j\omega n t}$$  (5)

In the case of a modulated signal, if $\omega_2$ is the carrier frequency and $\omega_1$ is the fundamental
frequency of the message or baseband signal, we have that $\omega_2 >> \omega_1$. In this situation, the
coefficients $\overline{X}_n(t)$, for $n = -\infty, \cdots, \infty$, can be seen as slow time-varying Fourier coefficients.
They are called complex envelopes of $x(t)$, and the expression (4) is called Envelope Fourier
series.

If we split $x(t)$ in two time axes, or dimensions, $t_1$ and $t_2$, its mathematical representation
can be rewritten as:

$$x(t_1, t_2) = \sum_{n=-\infty}^{\infty} \overline{X}_n(t_1) e^{j\omega_0 t_2}$$  (6)

where

$$\overline{X}_n(t_1) = \sum_{m=-\infty}^{\infty} X_{mn} e^{j\omega n t_1}$$  (7)

The signal $x(t_1, t_2)$ is $T_1$ periodic in $t_1$ and $T_2$ periodic in $t_2$, as shown in Fig. 1. Therefore,
the two-dimensional version of $x(t)$ has temporal dimensions associated with the time scales of
each of the fundamental frequencies. This is the principle of the multidimensional Fourier series
(in this case, the bidimensional Fourier series). Fig. 1 shows a bidimensional grid containing the
samples of $x(t)$ along the two time dimensions.
To represent the signal \( x(t_1, t_2) \) computationally, it is necessary to truncate the number of harmonics of the fundamental frequencies. Then, we can write:

\[
x(t_1, t_2) = \sum_{n=-N}^{N} \overline{X}_n(t_1) e^{j\omega_1 t_2} \tag{8}
\]

and

\[
\overline{X}_n(t_1) = \sum_{m=-M}^{M} X_{mn} e^{j\omega_m t_1} \tag{9}
\]

Thus, equations (8) and (9) present \( N \) harmonics of the carrier frequency and \( M \) harmonics of the baseband signal.

The Fourier coefficients of \( x(t) \) are shown in Fig. 2. These coefficients can be obtained by firstly applying unidimensional forward Fourier transforms along each row of the grid and then applying unidimensional forward Fourier transforms along the columns of the grid. This procedure corresponds to the bidimensional forward Fourier transform. However, when only the first part of the above procedure is performed, the time-varying coefficients \( \overline{X}_n(t_1) \), for \( n = -N, \ldots, N \), are found. It can be seen that the sampled signal \( \overline{X}_n(t_1) \) is taken at an instant \( t_1 = t_k \) and \( t_2 \) varying along one cycle of the high-frequency components. These components are called time-varying complex envelopes of \( x(t) \).

It can be noticed that the terms \( e^{j\omega_m t_1} \) are known at any instant \( t_2 \). However, only the terms \( \overline{X}_n(t_1) \) need to be known in order to represent those signals. These considerations are the
essence of the Nonlinear Envelope Method. Hence, any signal can be built by its complex envelopes when the signal is composed by low-frequency and high-frequency components. As the high-frequency time behavior is known, the signal \( x(t) \) can be reconstructed taking into account only the samples of its complex envelopes (low-frequency variations). Then, only a small number of samples (given by the sampling theorem) are necessary in order to represent an electric signal along one cycle of its low-frequency components.

\[
\text{Fig. 2 – Bidimensional grid of Fourier coefficients.}
\]

**Models for linear electrical elements**

By now, we are able to obtain some models for the basic linear elements of any electric circuit and also establish a way for evaluating the nonlinear functions of the nonlinear elements of circuits.

**Independent sources**

The standard form of an AM (Amplitude Modulation) and FM (Frequency Modulation) signal is:

\[
s(t) = f(t) \cos(\omega t + \varphi(t))
\]

where
• $f(t)$ is an amplitude modulating signal;
• $\varphi(t)$ is a phase modulating signal;
• $\frac{d\varphi(t)}{dt}$ is a frequency modulating signal;
• $\omega_2$ is the carrier frequency.

The function $s(t)$ can be conveniently written as:

$$s(t) = \frac{f(t)}{2} e^{-j\varphi(t)} e^{-j\omega_2 t} + \frac{f(t)}{2} e^{j\varphi(t)} e^{j\omega_2 t}$$

(11)

Using equation (8) and (9) to represent (11), we have:

$$s(t_1, t_2) = \sum_{n=-N}^{N} \overline{S}_n(t_1) e^{j\omega_2 t_2}$$

(7)

where

$$\overline{S}_n(t_1) = \sum_{m=-M}^{M} S_{nm} e^{j\omega_0 t_1}$$

(8)

It can be verified that in the above expressions the complex envelopes $\overline{S}(t_1)$, $n = -N, 0, \ldots, N$, are:

• $\overline{S}_{-1}(t_1) = \frac{f(t_1)}{2} e^{-j\varphi(t_1)}$ is the complex envelope representing the frequency $-\omega_2$;
• $\overline{S}_1(t_1) = \frac{f(t_1)}{2} e^{j\varphi(t_1)}$ represents the frequency $\omega_2$;
• and the rest of the envelopes are null.

The function $s(t)$ can represent any independent source waveform of an electric circuit. Hence, any independent source can be represented by two slow time-varying linear sources. This representation is shown in Fig. 3.

![Fig. 3 – Envelope Domain representation of an independent source.](image-url)
a) Linear Resistor

When expressions (4) and (5) are truncated and applied to the constitutive law of the linear resistor (12), and the resulting expression is split into two time dimensions $t_1$ and $t_2$, expression (13) can be obtained.

\[
v_R(t) = R i_R(t) \quad (12)
\]

\[
\sum_{n=-N}^{N} \bar{V}_{Rn}(t_1)e^{j\omega_0 t_2} = R \sum_{n=-N}^{N} \bar{I}_{Rn}(t_1)e^{j\omega_0 t_2} \quad (13)
\]

Since the equations of equality (13) are linearly independent, we can write for each harmonic of the fundamental frequency of the carrier:

\[
\bar{V}_{Rn}(t_1) = R \bar{I}_{Rn}(t_1) \quad (14)
\]

Expression (14) is true for $n = -N, \ldots, N$. Then, it can be verified that, for each harmonic frequency of $\omega_0$, there is an element composed by a resistance $R$. Therefore, the set of $2N + 1$ of those elements represents the resistor in the envelope domain. Fig. 4 shows the resistor model in the envelope domain.

![Fig. 4 – Envelope Domain representation of a linear resistor.](image)

b) Linear Capacitors

By applying truncated versions of expressions (4) and (5) to the linear capacitor constitutive law (15) and splitting the resulting expression in two time dimensions, one can obtain its equivalent representation in the envelope domain, as shown in Fig. 5.
\[ i_C(t) = C \frac{dv_C(t)}{dt} \]  

\[
\sum_{n=-N}^{N} I_{C_n}(t)e^{j\omega_n t} = C \frac{d}{dt} \left( \sum_{n=-N}^{N} V_{C_n}(t)e^{j\omega_n t} \right)
\]

Developing the equation above, taking the terms from both sides of this expression that represent the same frequency \( \omega_1 \), and splitting the resulting expression into two time dimensions \( t_1 \) and \( t_2 \), we obtain:

\[
\overline{I}_{C_n}(t_1) = C \frac{d\overline{V}_{C_n}(t_1)}{dt_1} + jn\omega_1 C \overline{V}_{C_n}(t_1)
\]

for \( n = -N, \ldots, N \). Then, it can be verified that, for each harmonic frequency of \( \omega_1 \), there is an element composed by a capacitance \( C \) in parallel with a complex conductance \( jn\omega_1 C \). Therefore, the set of \( 2N+1 \) of those elements represent the capacitor \( C \) in the envelope domain.

The procedure described above can be applied in order to obtain the inductor model. Doing so, the resulting expression is:

\[
\overline{V}_{L_n}(t_1) = L \frac{d\overline{I}_{L_n}(t_1)}{dt_1} + jn\omega_1 L \overline{I}_{L_n}(t_1)
\]
for \( n = -N, \ldots, N \). Therefore, to represent an inductor by the Nonlinear Envelope Method, we have a set of \( 2N + 1 \) elements. Each element is composed by an inductance \( L \) in series with a complex resistance \( jn\omega \). The envelope domain representation of an inductor is sketched in Fig.6.

![Envelope Domain representation of a linear inductor.](image)

Fig. 6 – Envelope Domain representation of a linear inductor.

**Models of nonlinear electric elements**

The models in the envelope domain of elements governed by nonlinear functions can also be easily obtained. The following sections present the models of nonlinear resistance/conductance and capacitance or charge in the envelope domain.

d) Nonlinear resistance/conductance

For nonlinear elements, we have a physical model described by a function with the form:

\[
q(t_1,t_2) = f_{NL}(g(t_1,t_2))
\]

where \( f_{NL}(\cdot) \) is a nonlinear operator. It is necessary to calculate the complex envelopes of \( q(t_1,t_2) \) using the time samples of \( g(t_1,t_2) \). Thus, it becomes necessary to obtain \( g(t_k,t_2) \) for \( t_1 = t_k \) and \( t_2 \) varying along one cycle of the high-frequency components of \( g(t) \). Those samples are obtained by applying the unidimensional inverse Fourier transform along one of the lines of the bidimensional grid of Fig. 2 where \( t_1 = t_k \). Doing so we obtain the vector of temporal samples presented in (20) from the complex envelope sample vector (19).

\[
\begin{bmatrix}
\overline{G}(t_k)
\end{bmatrix} = \begin{bmatrix}
\overline{G}_{-N}(t_k) & \ldots & \overline{G}_0(t_k) & \ldots & \overline{G}_N(t_k)
\end{bmatrix}
\]
To calculate the samples of \(q(t_1, t_2)\), the nonlinear operator \(f_{NL}(\cdot)\) is applied to the samples vector (20), leading to

\[
\mathbf{q}(t_k) = \left[ q(t_k, 0) \quad q(t_k, \Delta t_2) \quad \cdots \quad q(t_k, 2N\Delta t_2) \right]
\]

Finally, the unidimensional direct Fourier transform is applied to (21), resulting in:

\[
\overline{Q}(t_k) = \left[ \overline{Q}_{-N/2}(t_k) \quad \cdots \quad \overline{Q}_0(t_k) \quad \cdots \quad \overline{Q}_{N/2}(t_k) \right]
\]

Following this procedure, the complex envelopes of \(q(t_1, t_2)\) are obtained. This procedure can be summarized by the following expression:

\[
\overline{Q}(t_k) = \mathbf{F} \left\{ f_{NL} \left\{ \mathbf{F}^{-1} \left\{ \mathbf{G}(t_k) \right\} \right\} \right\}
\]

where \(\overline{Q}(t_k)\) is the vector of Fourier coefficients \(\mathbf{q}(t_k)\) at the time instant \(t_1 = t_k\), \(\overline{G}(t_k)\) is the vector of Fourier coefficients \(\mathbf{g}(t_k)\) at \(t_1 = t_k\), and \(\mathbf{F}\{\cdot\}\) and \(\mathbf{F}^{-1}\{\cdot\}\) are unidimensional direct and inverse Fourier transforms, respectively.

Through the procedure described above, the relationships between each envelope \(\overline{Q}_n(t_k)\) of \(q(t_1, t_2)\) with the envelopes \(\overline{G}_n(t_k)\) of \(g(t_1, t_2)\), for \(n = -N, ..., 0, ..., N\), are found. Hence, the way to describe the behavior of a nonlinear resistance/conductance in the envelope domain is known. It can be noticed that the element in the time domain corresponds to \(2N+1\) elements in the envelope domain and, as shown, the relationships between its electric signals are calculated using the unidimensional direct and inverse Fourier transforms.

e) Nonlinear capacitance (charge)

The formats of electric circuit equations commonly used by numeric simulators are expressed taking into account only currents and voltages [1,2,4]. Consequently, nonlinear capacitances (or charges) need special treatment to be included in those equations. In the case of \(q(t)\) being a nonlinear function of a voltage \(v(t)\), we have:

\[
q(t) = q(v(t))
\]

where

\[
q(t) = \sum_{n=-N}^{N} \overline{Q}_n(t)e^{j\omega_0 t}
\]

and
\[ v(t) = \sum_{n=-N}^{N} V_n(t)e^{jn\omega t} \]  

But,

\[ i(t) = \frac{dq(t)}{dt} \]  

where

\[ i(t) = \sum_{n=-N}^{N} I_n(t)e^{jn\omega t} \]  

Substituting expression (25) and (28) in (27), we obtain:

\[ \sum_{n=-N}^{N} I_n(t)e^{jn\omega t} = \frac{d}{dt} \left[ \sum_{n=-N}^{N} Q_n(t)e^{jn\omega t} \right] \]

Then,

\[ \sum_{n=-N}^{N} I_n(t)e^{jn\omega t} = \sum_{n=-N}^{N} \left[ jn\omega Q_n(t) + \frac{dQ_n(t)}{dt} \right] e^{jn\omega t} \]  

Therefore, when the signals are split into two time dimensions \( t_1 \) and \( t_2 \), it can be seen that:

\[ I_n(t_1) = jn\omega Q_n(t_1) + \frac{dQ_n(t_1)}{dt_1} \]  

for \( n = -N, \ldots, 0, \ldots, N \).

Consequently, it is possible to represent a nonlinear element of the type of charge (or capacitance) by the complex envelopes of the current that flows through it. It allows us to insert this type of element into the equations that represent an electric circuit. It must be seen that the coefficients \( Q_n(t_1) \), for \( n = -N, \ldots, 0, \ldots, N \), can be calculated using expression (31).

\[ \overline{Q}(t_1) = F\left\{ q\left( F^{-1}\{ \overline{V}(t_1) \} \right) \right\} \]  

Electric Circuit Equation Applied to the Envelope Method

In order to analyze the time behavior of a circuit, it is necessary to define the discrete models of circuit elements using one of the existing numerical integration methods [1,2]. After the discretization of the models, the equation system of expression (32) for \( t_1 = t_k \) can be built.
In expression (32), \( \overline{C}_{NL}(t_k) \), \( \overline{G}_{NL}(\overline{C}_{NL}(t_k)) \), and \( G_L(t_k) \) are, respectively, the Fourier coefficients of the nonlinear function commands \( c_{NL}(t_k,t_2) \), the nonlinear elements characterized by nonlinear functions \( g_{NL}(c_{NL}(t_k)) \), and the independent sources \( g_L(t) \). Those coefficients are present in the frequencies that are multiple of the carrier frequency. The matrices \( \hat{A}_1, \hat{A}_2 \) e \( \hat{A}_3 \) are composed by the models of the electric elements, obeying the circuit topology.

EHBSim solves equation (32) using the Newton-Raphson algorithm associated with a continuation technique, based on the control of the time-step size \( t_k - t_{k-1} \) [1,2].

III - EHBSim PROGRAM FUNCIONALITIES AND SIMULATION EXAMPLES

The EHBSim main screen, shown in Fig. 7, presents the circuit description as main component. As mentioned, EHBSim implements, besides the NLEM, the HBM. It allows one to make performance comparisons between both simulation methods.

The linear and nonlinear element models to be entered in the circuit description list follow a particular syntax. EHBSim provides, besides linear elements, nonlinear elements of current, charge, voltage, and flux types. The commands of those nonlinear elements can be voltages, delayed voltages, derivative voltages, currents, delayed currents, and derivative currents. The nonlinear functions that govern the relationships between the electric signals of nonlinear elements can be created by the user. The simulation results, required by the use of the probes V(Voltmeter) and A (Ammeter), are shown just after the end of the simulation. The frequency spectra of the responses, obtained by decomposition in Fourier series, as well as the waveforms are presented.

Fig. 7 – EHBSim main screen.
III.1 - Application examples

The aim of the following examples is to demonstrate the use of the EHBSim program. The examples also emphasize the superiority inherent to the NLEM when compared with the HBM for nonlinear circuits with multitone excitation. The computational costs compared are the time to achieve the circuit response and the amount of floating-point operations spent.

**FM Demodutador**

Fig. 8 depicts a FM demodulator. The excitation source $V_s$ provides a FSK (*Frequency Shift Keying*) signal with the following waveform:

$$ s(t) = A_s \cos(\omega_0 t + \int_0^t \theta(t) dt) $$

(33)

where $\theta(t)$ is a square wave with amplitude $\omega_c$, and $T$ is the $\theta(t)$ period. For this example, $A_s = 5 \text{ V}$, $\omega_0 = 2\pi \cdot 10^3 \text{ rad/s}$, $\omega_c = 2\pi \cdot 10^3 \text{ rad/s}$, and $T = 1 \text{ ms}$.

The steady-state response $V_1$ (voltage probe), obtained through the application of the NLEM, is shown in Fig. 9. Fig. 10 shows the difference between the solutions obtained by both simulation methods, where the precision of the NLEM is evident. The greater divergences are justified by the truncation of the frequency solution obtained by the HBM, which provides a temporal oscillation.
In Fig. 9, the points marked by squares represent the simulation instants performed by the NLEM. Thus, it can be seen that few simulation points are needed in order to obtain the circuit responses. Using traditional time domain integration techniques, it would be necessary many simulation points for each carrier cycle. The NLEM demands only the samples necessary to represent the complex envelopes of the modulated signals.

Table I shows the computational costs required by the HBM (second column), and by the NLEM (third column). In order to make the comparison, the circuit was simulated by NLEM until the steady-state response was reached.

Each row of the last column of Table I shows the ratio between the computational costs spent by the HBM and by the NLEM. The superiority of NLEM over HBM is quite impressive.
when the circuit is excited by modulated carriers. It is clear that the NLEM offers a much higher performance than the HBM.

Table I

<table>
<thead>
<tr>
<th></th>
<th>HBM</th>
<th>NLEM</th>
<th>Gain (HBM/NLEM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>1,998.90</td>
<td>13.63</td>
<td>147</td>
</tr>
<tr>
<td>Floating-point</td>
<td>6.166⋅10^9</td>
<td>8.277⋅10^7</td>
<td>75</td>
</tr>
<tr>
<td>operations</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**AM Modulator/Demodulator**

This example presents an AM modulator in cascade with an AM demodulator, as shown in Fig.11.

![Fig. 11 – AM modulator/demodulator.](image)

The excitation sources $V_{ft}$ and $V_{pt1}$ and $V_{pt2}$ have the waveforms of the expressions (34) and (35), respectively.

\[
v_{p}(t) = 0.5 \cos(2\pi 10^3 t) \text{ V}
\]

(34)

\[
v_{pt}(t) = 0.5 \cos(2\pi 10^3 t) \text{ V}
\]

(35)

The circuit responses, obtained by the use of the NLEM, are probed by the voltmeters $V_{1}$ (Fig. 12) and $V_{2}$ (Fig. 13).
Table II shows the performance measurements taken by the HBM and NLEM simulations.

<table>
<thead>
<tr>
<th></th>
<th>HBM</th>
<th>NLEM</th>
<th>Gain (HBM/NLEM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>11,245.00</td>
<td>132.31</td>
<td>85</td>
</tr>
<tr>
<td>Floating-point operations</td>
<td>$12.029 \times 10^{10}$</td>
<td>$6.634 \times 10^9$</td>
<td>18</td>
</tr>
</tbody>
</table>

The better performance of NLEM over HBM, when the circuit is excited by modulated carriers, is again evident.
Signal Amplifier

Another important advantage of the NLEM when it is compared with traditional methods is that it achieves the steady-state response quickly, i.e., predicts the long term transient behavior of circuits efficiently. This feature is due the main characteristic of this method: it handles only the slow time-varying complex envelopes of the modulated signals. Thus, only few samples of the complex envelopes along their cycles are needed. Simulating modulated circuits using traditional time domain integration methods would be highly computationally expensive because these methods use time steps calculated considering the highest frequency of the electrical signals and the simulation takes many cycles of the baseband signal. Thus, thousands of simulation points would be necessary. For example, consider the amplifier shown in Fig. 14. Fig. 15 shows the excitation signal, Fig. 16 shows the complete response of the amplifier for linear and saturation regions, and Fig. 17 shows the steady-state response for both operation regions. Three cycles of the baseband signal were necessary until the transient response vanished.

![Fig. 14 – Amplifier.](image)

![Fig. 15 – Excitation source V2.](image)
As shown if Figs. (15) and (16), the steady-state response is reached after three cycles of the baseband signal. These graphs also depict the relevant abrupt transition that occurs in the first cycle of the signals, the linearity of the response obtained in the linear case, and the high nonlinearity order of the response obtained in the saturation case.
Fig. 17 – Steady-state response \( V_1 \): (a) linear case, (b) saturation case.

Using traditional transient methods, about 2,000 points would be necessary in order to achieve the steady-state response in the first case (linear) and 5,000 simulation points in the
second case (saturation). The NLEM needs about 30 simulation points in both cases to achieve the steady-state response. It provides a medium gain of approximately 100. The greater the difference between the highest frequency of the modulated signals and the highest frequency of the baseband signals, the greater the gain provided by the NLEM.

IV - CONCLUSIONS

Due to the systematization implemented, the EHBSim program is capable to simulate circuit with any architecture of topology, strongly nonlinear, stable or unstable, and multi-excited by analog and digital signals. Original time integration algorithms were also implemented. These algorithms employ the continuation solutions concepts in order to guarantee secure and fast convergence. Since EHBSim implements the HBM and the NLEM algorithms, it was possible to make comparisons between the two techniques (numeric precision and computational effort). From the given examples, it was demonstrated the potentialities of NLEM for simulation of electric circuits excited by arbitrary modulated signals. NLEM also handles efficiently the transient response, which is extremely important to the analysis of digital communication systems.

REFERENCES