DIRECT SYNTHESIS OF MICROWAVE FILTERS USING MODIFIED SMALL REFLECTIONS THEORY

D. C. ALENCAR and L. R. A. X. MENEZES

Department of Electrical Engineering
University of Brasilia
e-mail: leonardo@ene.unb.br

Abstract

This work proposes a new design procedure of microwave filters using the theory of small reflections. The procedure is a simplification of the inverse scattering problem. The technique consists of determining the obstacle that generates the scattered field. In the case of microwave filters, the field is the time-domain input reflection coefficient and the geometry is the impedance profile of the filter. The procedure can be used to design different kinds of filters

Indexing terms: Filters, Reflection Coefficient

I. INTRODUCTION

The inverse scattering problem studies the reconstruction of the geometry of an object from its scattered field [1-4]. In the case of microwave passive linear devices, this field is the total reflection coefficient in time or frequency domains. Therefore, the inverse problem deals with the reconstruction of the impedance profile. The profile defines a non-uniform transmission line, which is determined from the desired reflection coefficient of the device. Consequently, this is a synthesis problem.

However, this problem usually does not have a unique solution. Therefore, some approximations are performed to obtain the desired impedance profile. This work uses some approximations based in the case of small spatial variations of the profile. In this case, the solution is obtained using the theory of small reflections [5] and by pre-distorting the reflection coefficient (renormalization). This approach enhances the applicability of the technique as a filter design tool.

II. THEORY

The geometry of the problem consists of a wave incident at the input point of a non-uniform transmission line of impedance profile $Z(x)$, Fig. 1. The input reflection coefficient in $x$, $\Gamma(x)$, is composed by several contributions of transmitted and reflected waves in the transmission line:
\[
\Gamma_i(x, 2\beta) = \frac{\Gamma(x) + \Gamma_i(x + \Delta x, 2\beta)e^{-2i\beta x}}{1 + \Gamma(x)\Gamma_i(x + \Delta x, 2\beta)e^{-2i\beta x}} \tag{1}
\]

Where: \(\Gamma_i(x)\) is the input reflection coefficient in \(x\) as a function of the electric length \(\Delta x\), \(\Gamma(x)\) is the local reflection coefficient, given by the local variation of the impedance profile, and \(\Gamma_i(x + \Delta x)\) is the input reflection coefficient in \(x + \Delta x\). If the impedance profile changes very slowly in space \(|\Gamma(x)|\) is very small:

\[
\Gamma_i(x, 2\beta) = \Gamma(x) + \Gamma_i(x + \Delta x, 2\beta)e^{-2i\beta x} \tag{2}
\]

Therefore, in a transmission line of length \(L\), the input reflection coefficient can be approximated by the Fourier transform of the local reflection coefficient along the line:

\[
\Gamma_i(0, 2\beta) = \int_0^L \Gamma(x)e^{-i2\beta x} dx \tag{3}
\]

If the impedance of the transmission line changes continuously along its length, the line can be approximated by a composition of infinitesimal sections of line with length \(\Delta x\) and impedance \(Z(x)\).

![Non uniform transmission line](image)

Fig. 1. Non uniform transmission line

In this kind of line the local reflection coefficient is:

\[
\Gamma(x) = \frac{Z(x + \Delta x) - Z(x)}{Z(x + \Delta x) + Z(x)} = \frac{dZ(x)}{2Z(x)} = \frac{1}{2}d(\ln Z(x)) \tag{4}
\]
Using (4) in (3) results in an expression describing the input reflection coefficient (x=0) as a function of the impedance profile of the transmission line:

\[
\Gamma_i(0,2\beta) = \frac{1}{2} \int_0^L e^{-j2\beta x} \frac{d}{dx} \ln(Z) \, dx
\]  

(5)

Equation (5) is the Fourier transform of the impedance profile function \(d(\ln Z)/dx\). This function is limited from zero to L. Therefore, it has value 0 if \(x<0\) or \(x>L\). Consequently, the inverse transform of (5) is an expression which describes the spatial variation of the impedance profile \(Z(x)\) as a function of the input reflection coefficient \(\Gamma_i(0,2\beta)\), [5].

\[
\frac{1}{2} \frac{d}{dx} \left( \ln(Z) \right) = \frac{1}{2\pi} \int e^{j2\beta x} \cdot \Gamma_i(0,2\beta) \cdot 2d\beta
\]  

(6)

Equation (6) is the synthesis equation. This is a direct result from the theory of small reflections. However, notice that (6) is true only for lengths between zero and L. In this synthesis procedure, the knowledge of the frequency or time domain behavior of the input reflection coefficient (design data) is used to generate the impedance profile that satisfies the requirements.

However, the approximations used in (2) and (4) do not guarantee that the created profile will satisfy the requirement of locally small reflection coefficients. In some cases, the product of \(\Gamma_i(x+\Delta x)\) by \(\Gamma(x)\) may not be small in all sections of the line. In other cases, secondary effects caused by the interaction of reflections in different sections in of line can result in low performance designs.

One solution to this problem is a pre-distortion of the total reflection coefficient at the input (3). The distortion is performed in a way that the local reflection coefficient remains small but the desired response is not corrupted. This is possible because the total reflection coefficient in passive transmission lines has the maximum absolute value of one.

This work used several renormalization techniques described in the literature [6]-[7]. The renormalization of the reflection coefficient tends to enhance its larger values and reduce lower values. Among the different techniques used, it was verified that the substitution:

\[
\Gamma_i(0,2\beta') = k[\Gamma_i(0,2\beta)]^n
\]  

(7)

results in a good compromise for \(k=3\) and \(n=2\).

The total size of the calculated device is related to Fourier transform on the synthesis equation (6). While the transformation is readily applicable, the calculated impedance profile is usually much longer than necessary. It can be truncated and still satisfy the design requirements. This is a result of the analysis equation (5). In general, wide-band devices result in smaller lengths of impedance profiles. The truncation of the filter depends on a reliable analysis procedure that...
can be used to validate the designs. Usually the Riccati equation [5] is used to analyze such problems. However, due to the nature of the truncation the unidimensional TLM method [8-9] can be used with advantages. The implementation procedure of the method introduces no dispersion associated with the numerical algorithm. Moreover, the associated Green's function of the TLM solution is a sampled version of the continuous one obtained using the wave equation. The calculated response is exact, except for the numerical precision of the computer.

In the TLM method, the device is discretized in N sections. Each one has a length $\Delta x$ and impedance $Z(x)$. The analysis is performed in the time-domain and at each timestep the reflected waves are calculated using:

$$V_{i} = \frac{2}{1+Y(x)}(Y(x)V_{k}^{i}(x) + V_{k}^{i}(x+\Delta x))$$

$$V_{k}^{i}(x) = V_{i} - V_{k}^{i}(x)$$

$$V_{k}^{i}(x+\Delta x) = V_{i} - V_{k}^{i}(x+\Delta x)$$

where $Y(x)$ is the ratio between admittances of adjacent section, $k$ is the timestep and $x$ is the position of the calculated section. The reflected waves are calculated using:

$$V_{k+1}^{i}(x) = V_{k}^{i}(x - \Delta x)$$

$$V_{k+1}^{i}(x + \Delta x) = V_{k}^{i}(x)$$

This version of TLM is slightly different from traditional implementations of the method. In this case, the method does not rely on the analogy between waves and fields, but in the direct application of the various reflections between adjacent sections. The calculation is performed in the time-domain for an appropriate number of timesteps. Once the simulation is completed, the frequency domain parameters are readily calculated.

The calculation of the impedance profile is not performed using (6), but by a combination of (4) and (6). The implementation is recursive and efficient. This is suitable due to the discretized nature of the Fourier transform calculation and of the TLM analysis procedure. The impedance profile is calculated from the inverse of (4) and the results of (6):

$$Z(x + \Delta x) = Z(x)\frac{1+\Gamma(x)}{1-\Gamma(x)}$$

III. EXPERIMENTAL RESULTS

The procedure was validated with two filter designs. In both cases, a MATLAB code was used to perform the synthesis procedure and generate an impedance profile file to be simulated in a TLM program. The design procedure is fully automatic for Thebyscheff or Butterworth filters. The
procedure handles pre-distortion and band corrections of the design algorithm. Once the profile is calculated, a TLM simulation is performed. Its results are used to determine the truncation and realization of the filter.

The designed filters were both third-order band reject filters with 0.5 dB ripple and 6GHz of central frequency. The filters had 41% and 11% fractional bandwidths. As expected the larger filter had the smaller bandwidth. The calculated impedance profile of one of the filters is shown in Fig. 2.

![Impedance profile of the band reject filter](image)

Fig. 2. Impedance profile of the band reject filter

The filters were assembled in stripline technology. The substrate had a dielectric constant $\varepsilon_r = 2.17$ and thickness of 1.524 mm. The filters were measured with the HP 8593E Spectrum analyzer. The results are shown in Fig. 3 and Fig.4. They are presented together with the simulated for comparison. The discrepancies between results are essentially caused by the technology used to assemble the filters. Due to the large size of the filters, it was not possible to achieve a uniform dielectric constant throughout the filter. The out of band oscillation is caused by the load used (measured return loss of -26 dB) which masked results below -20 dB.

The comparison between measured and simulated results shows good agreement in central frequency and desired fractional bandwidth (within 1%). The filters show the direct relationship between the bandwidth and impedance profile length. This is true in the band reject cases, which are easily described by its frequency domain characteristics. This suggests that this technique is suitable for the design of wide band devices. The truncation of the filter has some effects on its performance, especially in out-of-band characteristics. However, this is unavoidable because of the lengths involved.
IV. CONCLUSIONS

A new synthesis technique for microwave filters using modified theory of small reflections was presented. The procedure consists on using traditional theory of small reflections coupled with the pre-distortion of the input reflection coefficient. This renormalization procedure extends the scope of applicability of the design, while providing good results. Filters designed using this technique may not have an equivalent lumped circuit. The procedure was validated by TLM simulations and
experimental measurements. The results show good agreement (within 1%). New research is being pursued in the design of low and band pass filters.

REFERENCES

7. D. M. Pozar, Microwave Engineering, Addison-Wesley