Abstract— This paper presents a study on the performance of electro-optic (EO) modulators consisting of optical waveguides induced by residual thermal stress produced by silicon nitride (Si$_3$N$_4$) film deposited on bismuth germanate (Bi$_4$Ge$_3$O$_{12}$) substrate. Methodologies developed in previous works are used to analyze the performance related to electro-optic modulators. The analyses were performed by a full vector finite element method (FEM) based program, capable of multiphysics simulations, including evaluation of optical propagation characteristics of residual thermal stress-induced waveguides and main electro-optic parameters of EO modulators, considering different geometrical designs. The presented performance results (in terms of sizes of modes, effective refractive indices, characteristic impedances and electro-optic modulation depth) indicate the viability of EO modulators fabrication on cubic substrates with unusual directions of light propagation and electric field application.

Index Terms— integrated optic; optical waveguide; electro-optic modulator, FEM simulation.

I. INTRODUCTION

Electro-optical (EO) modulators in integrated optics are useful as components of optical sensor systems, especially in those that make use of white light interferometry (WLI) method. Several works describing WLI sensor systems applied as high voltage instrument transformers [1]-[3], for example, indicate advantages in using integrated optics electro-optical modulators as recover interferometer. In such cases it was verified that EO modulator should have no natural birefringence to achieve the best performance of system [3].

Integrated optical waveguides can be fabricated by many techniques as: ion diffusion, ion exchange, ion implantation, laser writing, thermo-optically-induced, stress-induced, residual thermal stress-induced, etc. Residual thermal stress-induced technique has the advantage of being compatible with microelectronics fabrication processes, allowing low cost and easily repeatable devices to be developed.

Residual thermal stress-induced integrated optical waveguides can be created modifying the
refraction index in predefined regions of the substrate. In cases where the substrate material is homogeneous, as crystals having cubic crystalline structure for instance, the induced waveguide will show no natural birefringence. In a former work, aiming to help the design of EO modulators on such a kind of substrates, a numerical modeling for two problems were presented: the stress-induced optical waveguides on Bi$_{12}$GeO$_{20}$ (a type of BGO) substrates with a layer of silicon oxide (SiO$_2$), and the quasi-static wave propagation (Transverse Electromagnetic Modes - TEM modes) of a coplanar stripline (CPS) with metallic electrodes of finite thickness [4]. In another paper, a particularly interesting case of orientation, in which the strain field is perpendicular to 110 crystallographic plane of Bi$_{12}$GeO$_{20}$ crystal, was presented [5]. Such a configuration allows the fabrication of an EO birefringence modulator with no polarization modes coupling, which is useful for WLI sensor systems.

Since Bi$_{12}$GeO$_{20}$ crystals exhibit optical activity, which is prejudicial to the efficiency of electro-optical modulation, the investigation of another material free of this effect was demanded. In this work the application of another type of BGO, the Bi$_4$Ge$_3$O$_{12}$, as substrate for EO modulators is studied. The EO modulator design, used in this study as model for simulations by FEM based program, is similar to the one previously proposed in former works [4], [5]. It consists of a BGO substrate, which dimensions (width and thickness) are considered to be infinites, covered by a Si$_3$N$_4$ thin film with symmetric metallic electrodes in the top, as shown in Fig. 1. In this figure, it is shown an expected guiding region located below the gap (G), which is symmetrical to the center of gap. The formation of a high refractive index region, depicted in Fig. 1 as guide region, is not obvious, but it will be demonstrated by numerical results in section V.

![Fig. 1. Electro-optic modulator built with Bi$_4$Ge$_3$O$_{12}$ substrate, Si$_3$N$_4$ buffer layer and two metallic electrodes.](image)

The structure shown in Fig. 1 is obtained in three steps: high temperature (at 750 °C) deposition of Si$_3$N$_4$ film, followed by, after cooling at 20°C, a lithography process where a small part of Si$_3$N$_4$ film is removed by etching, forming a gap region, and another lithography process where the electrodes deposition is done [6], [7]. At the end, a residual stress profile is obtained below the gap region,
induced by the different thermal expansion coefficient of BGO substrate and Si$_3$N$_4$ film. The residual thermal stress, distributed below the gap region, changes the refractive index of BGO by means of elastooptic effect, forming an integrated optical waveguide. The electrodes allow the application of modulating voltages and propagation of quasistatic electromagnetic waves (Transversal Electromagnetic Modes – TEM) along the waveguide. A modulation voltage applied to the electrodes creates a modulating electric field, which interacts with the optical guided wave via the linear electro-optic (Pockels) effect [8].

Table I presents the main material parameters of Bi$_4$Ge$_3$O$_{12}$ and Si$_3$N$_4$ used, respectively, as substrate and stressing layer of designed EO modulator.

| TABLE I. MAIN CHARACTERISTICS OF BI$_4$Ge$_3$O$_{12}$ AND SI$_3$N$_4$ |
|-----------------------------|-----------------------------|
| Crystal Class and Point Group | Cubic, 23 | Hexagonal (P6$_3$/m) |
| Density (kg/m$^3$) | 7130 | 3240 |
| Relative Dielectric Coefficient ($\varepsilon_r$) | 16 | 6-7 |
| Thermal Expansion Coefficient (K$^{-1}$) | $6.3 \times 10^{-6}$ | $2.8 \times 10^{-6}$ |
| Young’s Modulus (N/m$^2$) | $10.56 \times 10^{11}$ | $1.5 \times 10^{11}$ |
| Poisson’s Ratio | 0.26 | 0.24 |
| Strain-optic Coefficients | $p_{11} = 0.11$ | $p_{11} = 0.083$ |
| | $p_{12} = 0.083$ | $p_{12} = 0.083$ |
| | $p_{21} = 0.083$ | $p_{21} = 0.083$ |
| | $p_{44} = -0.0595$ |

II. STRESS ANALYSIS

For the stress analysis it was considered the propagation of light in a residual thermal stress-induced optical waveguide formed in a cubic 23 symmetry group crystal (the same as BGO). In the system of propagation axes shown in Fig. 2, $z'$ axis coincides with the direction of propagation of light and the $x'y'$ plane ($\overline{110}$), which is perpendicular to the direction of propagation, contains the cross section shown in Fig. 1.

![Fig. 2. Coordinate axes of the optical waveguide formed by residual thermal stress.](image)

The stress-strain relationship for isotropic material without initial stress is given by [9]:

\[ \sigma_{ij} = \epsilon_{ij} \mu \]
\[ T = D (S - \alpha \cdot \Delta T') \]  

where \( T \), \( D \), \( S \), \( \alpha \) and \( \Delta T' \) are the stress, the elasticity matrix, the strain, the thermal expansion coefficient and the temperature change (negative on cooling), respectively. The normal strain components and the shear strain components come from the global displacements \( (u, v, w) \) in the \( x \), \( y \) and \( z \) directions as:

\[
\begin{align*}
S_1 &= \frac{\partial u}{\partial x},
S_2 &= \frac{\partial v}{\partial y},
S_3 &= \frac{\partial w}{\partial z},
S_4 &= \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right),
S_5 &= \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right),
S_6 &= \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)
\end{align*}
\]

The plane strain condition prevails in geometries whose size in the propagation direction is large when compared to dimensions in directions that are parallel to the cross section. In the plane strain assumption \( S_3, S_4 \) and \( S_5 \) become zero.

The photoelastic effect, also known as elasto-optic effect, in a material couples the mechanical strain to the optical index of refraction (\( n \)). This effect can be described using the contracted indices notation by [10]:

\[
\Delta \left( \frac{1}{n^i} \right) = p_{ij} \cdot S_j, \quad i, j = 1, 2, ..., 6
\]

where \( p_{ij} \) represents the strain-optic tensor.

The index ellipsoid equation of Bi\(_4\)Ge\(_3\)O\(_12\) in the presence of a strain can be written as:

\[
x^2 \left( \frac{1}{n_{BiGeO_{12}}^2} + p_{11} S_1 + p_{12} S_2 \right) + y^2 \left( \frac{1}{n_{BiGeO_{12}}^2} + p_{12} S_1 + p_{11} S_2 \right) + 2xy (p_{44} S_6) = 1
\]

The new diagonal principal axes of Bi\(_4\)Ge\(_3\)O\(_12\) are obtained by rotating the old principal axes as indicated in Fig. 2 and are given by:

\[
n_x' = n_{x BGO} - \frac{1}{2} \sqrt{2n_{x BGO}^3} \left( p_{11} + p_{12} \right) \cos^2 \beta - \frac{1}{2} \sqrt{2n_{x BGO}^3} \left( p_{12} + p_{11} \right) \cos^2 \alpha \cdot \sin^2 \beta
- \frac{1}{2} \sqrt{2n_{x BGO}^3} \left( p_{12} + p_{11} \right) \sin^2 \alpha \cdot \sin^2 \beta + \frac{1}{2} \sqrt{2n_{x BGO}^3} p_{44} \sin 2\beta \cdot \cos \alpha
\]

\[
n_y' = n_{y BGO} - \frac{1}{2} \sqrt{2n_{y BGO}^3} \left( p_{11} + p_{12} \right) \cos^2 \alpha - \frac{1}{2} \sqrt{2n_{y BGO}^3} \left( p_{12} + p_{11} \right) \sin^2 \alpha
- \frac{1}{2} \sqrt{2n_{y BGO}^3} p_{44} \sin 2\alpha
\]

\[
n_z' = n_{z BGO}
\]
\[ n_z = n_{z\text{BGO}} - \frac{1}{2} \sqrt{2n_{z\text{BGO}}}^3 \left( p_{11} + p_{12} \right) \sin^2 \beta - \frac{1}{2} \sqrt{2n_{z\text{BGO}}}^3 \left( p_{12} + p_{11} \right) \cos^2 \alpha \cdot \cos^2 \beta \]
\[ - \frac{1}{2} \sqrt{2n_{z\text{BGO}}}^3 \left( p_{11} + p_{12} \right) \sin^2 \cdot \cos \beta - \frac{1}{2} \sqrt{2n_{z\text{BGO}}}^3 p_{44} \sin 2\beta \cdot \cos \alpha \]
\[ + \frac{1}{2} \sqrt{2n_{z\text{BGO}}}^3 p_{44} \sin \alpha \cdot \cos \beta \] (5c)

where \( \alpha \) and \( \beta \) are equal to \( \frac{\pi}{4} \).

### III. ELECTRO-OPTICAL ANALYSIS

Coplanar striplines without metallic shielding (open boundary) support propagation of quasi-TEM modes that are related to the solutions of the Laplace equation for the electric potential \( \phi \):
\[ \nabla \cdot \left( \varepsilon_r \cdot \nabla \phi \right) = 0 \] (6)
where \( \varepsilon_r \) represents the relative permittivity tensor.

The electrical wave propagating through the CPS waveguide establishes an electric field profile on the dielectric material (substrate) that provokes changes in the refractive index (via electro-optic effect). Considering the linear electro-optic effect (Pockels effect) it’s possible to describe the changes in the impermeability tensor, in the contracted indices representation, as [10]:
\[ \Delta n_{ij} = \Delta \left( \frac{1}{n_j^2} \right) = r_{ij} \cdot E_j \] (7)
where \( r_{ij} \) are the electro-optic tensor elements and \( E_j \) represents the transversal electric field components (\( E_x \) and \( E_y \)) of modulation signal.

For the \( \text{Bi}_4\text{Ge}_3\text{O}_{12} \) substrate, the perturbed equation of the index ellipsoid in the presence of a modulating electric field \( E \) can be represented as:
\[ n_x = \frac{3}{2} \sqrt{2} n_x n_y \left( - \sqrt{2} n_x n_y^3 \cos^2 \theta - \sqrt{2} n_x^3 n_y \sin^2 \theta - \sqrt{2} n_x^3 n_y^3 r_{11\text{BGO}} E_x \sin^2 \theta \right) \]
\[ - n_x^3 n_y r_{11\text{BGO}} E_y \sin^2 \theta \] (8a)
\[ n_y = \frac{3}{2} \sqrt{2} n_x n_y \left( - \sqrt{2} n_x n_y^3 \sin^2 \theta - \sqrt{2} n_x^3 n_y \cos^2 \theta - \sqrt{2} n_x^3 n_y^3 r_{11\text{BGO}} E_x \cos^2 \theta \right) \]
\[ - n_x^3 n_y r_{11\text{BGO}} E_y \sin^2 \theta \] (8b)
\[ n_z = n_z + \frac{1}{2} n_z^2 r_{11\text{BGO}} E_x \] (8c)
The residual thermal stress on Bi$_4$Ge$_3$O$_{12}$ substrate allows the fabrication of integrated optical waveguides that support modes with low propagating losses. Submitting such a kind of waveguide to an external electric field is possible to modulate the propagation phase of each polarization mode. Considering the elastooptic and electro-optic effects, the refractive indexes of the BGO substrate can be represented as:

$$n''_{\text{BGO}} = n_{\text{BGO}}^{\text{bulk}} + \Delta n_{\text{BGO Strain}} + \Delta n_{\text{BGO Pockels}}$$  (9)

IV. OPTICAL MODE ANALYSIS

An optical channel waveguide supports the propagation of two types of modes: E$^x$ and E$^y$ modes. The modal characteristics can be evaluated by solving the vectorial Helmholtz equation in the source-free assumption:

$$\nabla \times \mu^{-1} \nabla \times E - k_0^2 \varepsilon_r E = 0$$  (10)

where $\varepsilon_r=\varepsilon_0 n^2$ is the electric permittivity and $E$ represents the modal electric field. In optical analyses $n$ is considered as the refractive indexes after the changes caused by the strain-optic and electro-optic effects.

V. NUMERICAL RESULTS

The numerical analyses were made with the aid of a commercial finite element analysis, solver and simulation software called COMSOL$^\text{®}$ [11], which is a software environment able to perform design, modeling and simulation of physics-based systems, including engineering and scientific applications involving weakly coupled phenomena, or multiphysics, like stress, electric and optical fields. The precision of all results obtained from simulations using COMSOL$^\text{®}$ program was specified to 6 decimal positions.

The studies consider a strain-induced optical waveguide formed on Bi$_4$Ge$_3$O$_{12}$ substrate coated with a stress applying Si$_3$N$_4$ film, as shown in Fig. 1. The Fig. 3 shows typical $S_1$, $S_2$ and $S_6$ strains profiles obtained by simulations in the induced waveguide region.
Fig. 3. Strain-induced profiles in the optical waveguide region. (a) Strain S₁ (b) Strain S₂ (c) Strain S₆.

The quasi-static analysis allows obtaining the electric potential profile and then the modulating electric field distribution. The Fig. 4 presents the electric potential isolines and the arrows represent the electric field of the modulation wave.

Fig. 4. Isolines of electric potential and electric field (arrows) of TEM wave. View of the gap region.

The Fig. 5 presents a typical optical electric field profile of Eₓ and Eᵧ modes, the guide is single-mode with stress-induced birefringence of about $\sim 10^{-4}$ for all geometric configuration analyzed.
The Fig. 6 shows the effective index $N_{\text{eff}}$ for the $E^x$ modes of the residual thermal stress induced waveguide as a function of the gap in the Si$_3$N$_4$ film for different film thickness. When the gap increases, $N_{\text{eff}}$ decreases for a Si$_3$N$_4$ film thickness of 500 nm at a temperature of 750°C.

From the sequence of graphs a, b, c and d of Fig. 6, it is possible to observe that, for each gap width, when wavelength increases both effective indexes (modes $E^x$ and $E^y$) decrease. Also it is possible to notice that such decreasing is larger for the smaller gaps. As result of this behavior the maximum value of effective index is progressively changing its position to the higher values of gap.

The description of EO modulators based on traveling waves, with no loss and fabricated with CPS electrodes deposited parallel to the channel waveguide, is made by means of their main parameters, which are: the impedance ($Z_c$), the electrical wave effective index ($N_{\text{eff}}$), the product bandwidth length ($\Delta f \cdot L$) and the overlap integral ($\Gamma$), defined as [12]:

\begin{align}
Z_c &= \frac{1}{c} \sqrt{\frac{1}{C \cdot C_1}} \\
N_{\text{eff}} &= \sqrt{\frac{C}{C_1}} \\
\Delta f \cdot L &= \frac{1.4 \cdot c}{\pi (N_{\text{eff}} - n_{\text{eff}})} \\
\Gamma &= \frac{G}{\Delta V} \int \int E_{\text{op}}^2 E_{\text{TEM}}^2 \, dx \, dy \\
&= \int \int E_{\text{op}}^2 \, dx \, dy
\end{align}

where $c$ is the free-space light velocity, $C$ is the capacitance per unit length of the CPS considering the dielectric materials, $C_1$ is the capacitance per unit length of the CPS in vacuum, $n_{\text{eff}}$ is the effective index of optical mode, $E_{\text{TEM}}$ are the transversal electric field components, $E_{\text{op}}$ is the modal optical electric field, $G$ is the gap width, and $\Delta V$ is the voltage difference between electrodes.
Fig. 6. Effective index ($N_{eff}$) of guided fundamental optical modes as function of gap width (a) $\lambda = 0.633$ $\mu$m (b) $\lambda = 0.950$ $\mu$m (c) $\lambda = 1.310$ $\mu$m (d) $\lambda = 1.550$ $\mu$m.

For each waveguide, considering sinusoidal electric wave propagation, such parameters are determined in this work using COMSOL® program by a study called optical and microwaves waveguides analysis.

The Fig. 7 presents the characteristic impedance, $Z_c$, and the effective index, $N_{eff}$, to the electric modulation wave as functions of the gap width, G, considering Si$_3$N$_4$ film with 500 nm of thickness and wavelengths, $\lambda$, of 0.633 $\mu$m, 1.310 $\mu$m and 1.550 $\mu$m. For $\lambda = 0.633$ $\mu$m, $N_{eff}$ decreases and $Z_c$ increases as the gap width, G, increases. For $\lambda = 1.310$ $\mu$m, $N_{eff}$ decreases and $Z_c$ slightly increases as the gap width, G, increases. The last behavior is also observed for $\lambda = 1.550$ $\mu$m.

For all wavelengths and $G = 5$ $\mu$m, $Z_c$ is about 54 $\Omega$ and $N_{eff}$ varies in the range of 2,032 to 2,063. The resulting $N_{eff}$ allows satisfactory impedance matching with external modulation source, but the bandwidth will be affected and limited by the difference between $N_{eff}$ and $n_{eff}$, which varies with $\lambda$.

The Fig. 8 shows the overlap integral factor as function of electrode thickness for each wavelength, $\lambda$, of 0.633 $\mu$m, 1.310 $\mu$m and 1.550 $\mu$m.
Based on overlap integral factor values the half-wave voltage, $V_{\pi}$, of the EO modulator can be calculated. Results for such calculation will be presented in future work.

Fig. 7. Characteristic impedance and effective index of electrical wave as functions of the gap width, in the Si$_3$N$_4$ thin film, considering the metallic electrodes and the film have thicknesses of 500 nm (a) $\lambda = 0.633 \, \mu$m (b) $\lambda = 1.310 \, \mu$m (c) $\lambda = 1.550 \, \mu$m.

Fig. 8. Overlap integral as function of the gap width (a) $\lambda = 0.633 \, \mu$m (b) $\lambda = 1.310 \, \mu$m (c) $\lambda = 1.550 \, \mu$m.
VI. CONCLUSIONS

Residual stress-optical and electro-optical analyses of a modulator built in Bi$_2$Ge$_3$O$_{12}$ substrate, with a buffer layer of Si$_3$N$_4$ and two metallic electrodes of finite thickness in a CPS configuration is presented. As reported previously in former works [4], [5], it is demonstrated that the stress-induced channel optical waveguide operates in single mode and support guided modes for the specific geometrical configuration proposed.

The study of some characteristics of EO modulators ($Z_c$, $N_{eff}$ and $\Gamma$) as functions of a main geometric parameter $G$, shows that it is possible to obtain a suitable set of characteristics for an useful real device and aiding the fabrication process of EO modulators by, for instance, reducing the number of iterations of this process.

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