Abstract

A power flow equation describing the power flow of electromagnetic waves in a real multimode waveguide represented by a linear multichannel transmission system exhibiting an attenuation, dispersion, and an interchannel interaction is solved in three iterative ways. In the case of constant attenuation and constant propagation velocities in all channels and for the convolution interchannel interaction, a closed analytical solution of the problem is presented. Interesting forms of the solution, transfer function, and impulse response of the system which enable us to separate and compare the attenuation, dispersion, and coupling effects independently of each other are derived. The transfer function, the impulse response, and excitation conditions for their measurements are further discussed. Relations which make it possible to compare different experimental data obtained under specific excitation conditions in various laboratories are determined. Finally, a linear system with memory at time is studied and it is shown that the real multimode waveguide as a linear multichannel transmission system can be considered as a general linear system with memory at space.

1 Introduction

Multimode waveguides (both microwave and optical) can transmit a great number of guided modes. As a result of losses in real media, the modes may be attenuated, or amplified (as a result of gain) in active media. During propagation, they are also affected by the modal or chromatic (material and waveguide) dispersion, i.e., the mode propagation velocity and the propagation velocities of the individual spectral components of each mode depend on the frequency. In a real multimode waveguide in which
stochastic perturbations of the waveguide geometry or refractive index fluctuations of the waveguide medium are presented, a mutual power interaction (mode conversion, mode coupling) can occur among the individual guided modes. Such waveguide represents a rather complicated multichannel transmission system.

2 Power flow equation

Let us consider an \( N \)-mode waveguide. A power flow in this waveguide is described by the set of \( N \) coupled first-order differential equations [1]

\[
\frac{\partial p_m(z,t)}{\partial z} + \frac{1}{v_m} \frac{\partial p_m(z,t)}{\partial t} = -\alpha_mp_m(z,t) + \sum_{n=1}^{N} h_{nm}[p_n(z,t) - p_m(z,t)],
\]

\[m = 1, \ldots, N, \tag{1}\]

where \( p_m(z,t) \) is the power of the \( m \)-th mode along the waveguide (coordinate \( z \)) at instant \( t \), \( v_m \) is the propagation velocity of the \( m \)-th mode, \( \alpha_m \) is its attenuation coefficient, and \( h_{nm} \) is the power coupling coefficient between modes \( n \) and \( m \).

For the multimode waveguide with a sufficiently large value of \( N \), the discrete spectrum of the guided modes can be replaced by a quasi-continuum, and consequently, instead of (1), a single integrodifferential equation can be written [2]

\[
\frac{\partial p(\theta, z, t)}{\partial z} + \frac{1}{v(\theta)} \frac{\partial p(\theta, z, t)}{\partial t} = -\alpha(\theta)p(\theta, z, t) + \int_{0}^{\Theta} c(\phi, \theta)[p(\phi, z, t) - p(\theta, z, t)]d\phi,
\]

\[\theta \in (0, \Theta), \tag{2}\]

where \( \theta \) and \( \phi \) are, in the ray optics approximation, the propagation directions of two guided modes, \( \Theta \) is the critical angle of the guided modes, and \( c(\phi, \theta) \) is the two-dimensional coupling function expressing some coupling between modes \( \phi \) and \( \theta \).

2.1 Iterative solution of the power flow equation

Let us introduce the time Fourier transform \( P(\theta, z, \omega) \) of the power \( p(\theta, z, t) \)

\[
P(\theta, z, \omega) = \int_{-\infty}^{\infty} p(\theta, z, t)e^{-i\omega t}dt,
\]

\[\tag{3}\]

where \( \omega \) is the angular frequency, and the inversion formula

\[
p(\theta, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\theta, z, \omega)e^{i\omega t}d\omega.
\]

After substitution (3) into (2), we get

\[
\frac{\partial P(\theta, z, \omega)}{\partial z} + \gamma(\theta, \omega)P(\theta, z, \omega) = \int_{0}^{\Theta} c(\phi, \theta)[P(\phi, z, \omega) - P(\theta, z, \omega)]d\phi,
\]

\[\tag{5}\]

where

\[
\gamma(\theta, \omega) = \alpha(\theta) + \frac{i\omega}{v(\theta)}.
\]

\[\tag{6}\]

Eq. (5) describes the problem in the frequency domain. In this section, three approximate techniques for the solution of (5) based on iterative methods will be summarized.
1. Let us suppose
\[ P(\theta, z, \omega) = \sum_{l=0}^{\infty} b_l(\theta) z^l. \] (7)

Inserting (7) into (5) and after some manipulations, one can obtain a recurrence relation
\[ b_l(\theta) = \frac{1}{l} \left\{ \int_0^\theta c(\phi, \theta) [b_{l-1}(\phi) - b_{l-1}(\theta)] d\phi - \gamma(\theta, \omega) b_{l-1}(\theta) \right\}, \] (8)

where \( l = 1, 2, \ldots \), and \( b_0(\theta) = P(\theta, 0, \omega) \) is the input excitation of the waveguide at \( z = 0 \). Thus, for a given input power excitation, recurrence relation (8) makes it possible to find the solution of the problem.

2. In the case of no conversion, \( c(\phi, \theta) = 0 \), and (5) provides a zero-order approximation
\[ P_0(\theta, z, \omega) = P(\theta, 0, \omega) e^{-\gamma(\theta, \omega) z}. \] (9)

In the following step, we solve the equation
\[ \frac{\partial P_1(\theta, z, \omega)}{\partial z} + \gamma(\theta, \omega) P_1(\theta, z, \omega) = f_0(\theta, z), \] (10)

where
\[ f_0(\theta, z) = \int_0^\theta c(\phi, \theta)[P_0(\phi, z, \omega) - P_0(\theta, z, \omega)] d\phi. \] (11)

Thus, in the first-order approximation, we have the solution
\[ P_1(\theta, z, \omega) = P_0(\theta, z, \omega) + \int_0^z f_0(\theta, \eta) e^{-\gamma(\theta, \omega)(z-\eta)} d\eta. \] (12)

By analogy, we get for the \( l \)-th approximation
\[ P_l(\theta, z, \omega) = P_0(\theta, z, \omega) + \int_0^z f_{l-1}(\theta, \eta) e^{-\gamma(\theta, \omega)(z-\eta)} d\eta, \] (13)

where
\[ f_{l-1}(\theta, \eta) = \int_0^\theta c(\phi, \theta)[P_{l-1}(\phi, z, \omega) - P_{l-1}(\theta, z, \omega)] d\phi. \] (14)

Recurrence relation (13) [as well as (8)] enables us to obtain the power flow along the waveguide. But now, we find the power flow equation solution neglecting the mode coupling along the waveguide firstly, and in the following steps, we correct this solution by including the mode interaction. Both procedures are convenient for numerical calculations [3].
3. Still another approximate technique based on solving the Fredholm’s integral equation may be used [4]. Eq. (5) can be rewritten in the following way

\[ P(\theta, z, \omega) - \int_0^\Theta K(\phi, \theta, \omega) P(\phi, z, \omega) d\phi = g(\theta, z, \omega), \]

where

\[ K(\phi, \theta, \omega) = \frac{c(\phi, \theta)}{\gamma(\theta, \omega)} \]

and

\[ g(\theta, z, \omega) = -\frac{1}{\gamma(\theta, \omega)} \frac{\partial P(\theta, z, \omega)}{\partial z}. \]

Let us take \( P_0(\theta, z, \omega) \) given by (9) as the zeroth-order approximation and then solve the integral equation

\[ P_1(\theta, z, \omega) - \int_0^\Theta K(\phi, \theta, \omega) P_1(\phi, z, \omega) d\phi = g_0(\theta, z, \omega), \]

where

\[ g_0(\theta, z, \omega) = -\frac{1}{\gamma(\theta, \omega)} \frac{\partial P_0(\theta, z, \omega)}{\partial z}. \]

The solution of (18) can be written in the form [5]

\[ P_1(\theta, z, \omega) = \sum_{j=0}^{\infty} K^j g_0(\theta, z, \omega), \]

where \( K \) is the Fredholm’s operator for the integral kernel \( K(\phi, \theta, \omega) \)

\[ K^j g(\theta, z, \omega) = \int_0^\Theta \int_0^\Theta \ldots \int_0^\Theta K(\theta, \theta_1, \omega) K(\theta_1, \theta_2, \omega) \ldots K(\theta_{j-1}, \theta_j, \omega) \times \]

\[ \times g(\theta_j, z, \omega) d\theta_1 d\theta_2 \ldots d\theta_j \]

and \( K^0 g(\theta, z, \omega) = g(\theta, z, \omega) \). Finally, we have for the \( l \)-th approximation

\[ P_l(\theta, z, \omega) = \sum_{j=0}^{\infty} K^j g_{l-1}(\theta, z, \omega), \]

where \( l = 1, 2, \ldots, \) and

\[ g_{l-1}(\theta, z, \omega) = -\frac{1}{\gamma(\theta, \omega)} \frac{\partial P_{l-1}(\theta, z, \omega)}{\partial z}. \]

We may note that this approximate technique is rather more complicated than both the previous ones.
Note. By means of these techniques, one can determine the power flow through the waveguide system with a required accuracy. For the waveguide excitation \( p(\theta,0,t) = p_0(\theta)\delta(t) \), where \( \delta(t) \) is the Dirac’s function, we have \( P(\theta,0,o) = p_0(\theta) \). The ratio \( H(\theta,z,\omega) = P(\theta,z,\omega)/p_0(\theta) \) is often called as the transfer function and its inverse Fourier transform \( h(\theta,z,t) \) as the impulse response of the system [6]. But it is necessary to say that this is not quite correct. Namely, as one can see, these functions depend on the system excitation. Their measurements can be usually done in three ways: 1. to excite the single mode \( [p_0(\theta) = \delta(\theta - \theta_0)] \), particularly \( \theta_0 = 0 \); 2. to excite all the modes uniformly \( [p_0(\theta) = 1] \); or 3. to assure a so-called steady state power distribution (SSPD) at the input of the waveguide [7]. For example, let us consider the uniform excitation of all the modes and follow the second of the three above-described iterative techniques. Then, we have for the zeroth-order approximations \( H_0(\theta,z,\omega) = \exp[-\alpha(\theta)z] \exp[-i\omega z/v(\theta)] \) and \( h_0(\theta,z,t) = \exp[-\alpha(\theta)z|\delta[t - z/v(\theta)] \]. Of course, these approximations characterize the waveguide in which mode coupling phenomena are not presented. If we include the cross-mode interaction, we can derive the first approximations

\[
H_1(\theta,z,\omega) = H_0(\theta,z,\omega) + \int_0^\Theta c(\phi,\theta) \left[ e^{-\gamma(\phi,\omega)z} - e^{-\gamma(\theta,\omega)z} \right] \frac{\gamma(\theta,\omega) - \gamma(\phi,\omega)}{\gamma(\theta,\omega) - \gamma(\phi,\omega)} + ze^{-\gamma(\theta,\omega)z} \right] d\phi \quad (24)
\]

and

\[
h_1(\theta,z,t) = h_0(\theta,z,t) \left[ 1 - z \int_0^\Theta c(\phi,\theta)d\phi \right] + \int_0^\Theta \frac{c(\phi,\theta)}{1/v(\theta) - 1/v(\phi)} \times
\]

\[
\left\{ e^{-a(\theta,\phi)z} - e^{-a(\phi,\theta)z} \right\} d\phi , \quad (25)
\]

where

\[
a(\theta,\phi) = \frac{\alpha(\theta) - \alpha(\phi)}{1/v(\theta) - 1/v(\phi)} > 0 \quad \text{and} \quad t > \frac{z}{v(\phi)}, \quad \frac{z}{v(\theta)}. \quad (26)
\]

2.2 Analytical solution of the power flow equation

The input of the waveguide depends generally on two arguments \( \theta \) and \( t \). Thus, to find the transfer function and the impulse response of the waveguide it is necessary to solve the problem by means of two-dimensional Fourier transform

\[
P(\Omega,z,\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(\theta,0,t) e^{-i(\Omega \theta + \omega t)},
\]

\[
p(\theta,0,t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(\Omega,z,\omega) e^{i(\Omega \theta + \omega t)}. \quad (27)
\]

Let us rename

\[
c(\phi,\theta) = \left\{ \begin{array}{ll}
c(\phi,\theta) & \forall \phi \in (0,\Theta) \\
0 & \text{elsewhere}
\end{array} \right.
\]

and consider

\[
\alpha(\theta) = \sum_{i=0}^{\infty} \alpha_i \theta^i \quad \text{and} \quad \frac{1}{v(\theta)} = u(\theta) = \sum_{i=0}^{\infty} u_i \theta^i. \quad (29)
\]
Further, if we assume function $c(\phi, \theta)$ in the following form
\[ c(\phi, \theta) = c(\theta - \phi), \] (30)
we can write, after substitution (27) into (2) and using (28) and (29), a rather complicated partial differential equation
\[ \frac{\partial P(\Omega, z, \omega)}{\partial z} + \left[ \beta_0 + i\omega u_0 - C(\Omega) \right] P(\Omega, z, \omega) + \sum_{i=1}^{\infty} (\beta_i + i\omega u_i) \frac{\partial^i P(\Omega, z, \omega)}{(-i)^i \partial \Omega^i} = 0, \] (31)
where $C(\Omega)$ is given by the one-dimensional Fourier transform of function $c(\xi)$, i.e.,
\[ C(\Omega) = \int_{-\infty}^{\infty} c(\xi) e^{-i\Omega\xi} d\xi. \] (32)
Only in the case of constant $v(\theta) = v_0$ and $\alpha(\theta) = \alpha_0$, one is able to get an analytical solution of the problem. Instead of (31), we have
\[ \frac{\partial P(\Omega, z, \omega)}{\partial z} + \left[ \beta_0 + \frac{i\omega}{v_0} - C(\Omega) \right] P(\Omega, z, \omega) = 0, \] (33)
which has the solution
\[ P(\Omega, z, \omega) = P(\Omega, 0, \omega) e^{-[\omega_0 + \frac{i\omega}{v_0} + f(\Omega)]z}, \] (34)
where
\[ f(\Omega) = \int_{-\infty}^{\infty} c(\xi) \left( 1 - e^{-i\Omega\xi} \right) d\xi. \] (35)
By means of (27), we finally have in the time domain
\[ p(\theta, z, t) = e^{-\omega_0 z} p_f(\theta, z, t), \] (36)
where $p_f(\theta, z, t)$ is given by the one-dimensional inverse Fourier transform of function $\tilde{p}(\Omega, 0, t - z/v_0) \exp[-f(\Omega)z]$, i.e.,
\[ p_f(\theta, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{p}(\Omega, 0, t - \frac{z}{v_0}) e^{-f(\Omega)z} e^{i\Omega\theta} d\Omega, \] (37)
and where $\tilde{p}(\Omega, 0, t - z/v_0)$ is given by the one-dimensional Fourier transform of the time shifted input excitation, i.e.,
\[ \tilde{p}(\Omega, 0, t - \frac{z}{v_0}) = \int_{-\infty}^{\infty} p(\psi, 0, t - \frac{z}{v_0}) e^{-i\Omega\psi} d\psi. \] (38)
Hence
\[ p(\theta, z, t) = \frac{e^{-\omega_0 z}}{2\pi} \int_{-\infty}^{\infty} p(\psi, 0, t - \frac{z}{v_0}) \int_{-\infty}^{\infty} e^{-f(\Omega)z+i\Omega(\theta-\psi)} d\Omega d\psi. \] (39)
2.3 The transfer function and the impulse response of the multimode waveguide structure

In the note at the end of Sect. 2.1, we discussed "the transfer function" and "the impulse response" of the real multimode waveguide. It has been shown that for the exact definition of these function, the system must be considered as the two-dimensional one. By means of (34), we straightforwardly get the transfer function

\[
H(\Omega, z, \omega) = \frac{P(\Omega, z, \omega)}{P(\Omega, 0, \omega)} = e^{-\alpha_0 z} \cdot e^{-i\omega_0 z} \cdot e^{-i\Omega z}.
\]  

(40)

Then, using the two-dimensional Fourier transform (27), we can obtain the impulse response of the system in the form

\[
h(\theta, z, t) = e^{-\alpha_0 z} \cdot \delta \left(t - \frac{z}{v_0}\right) \cdot f_{\epsilon}(\theta, z),
\]

(41)

where \(f_{\epsilon}(\theta, z)\) is given by the one-dimensional inverse Fourier transform of the function \(\exp[-f(\Omega)z]\), i.e.,

\[
f_{\epsilon}(\theta, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\Omega z} e^{i\Omega \theta} d\Omega.
\]

(42)

As it can be seen from the forms of (40) and (41), the transfer function and the impulse response of the multimode waveguide are given as the products of three terms each of them represents the influence of the losses, the mode propagation velocity, and the mode coupling separately.

2.4 Measurement of the transfer function and the impulse response

Let us consider the input waveguide excitation

\[
p(\theta, 0, t) = \delta(\theta)\delta(t),
\]

(43)

which represents the excitation of a very short and sufficiently intense pulse in the fundamental mode \((\theta = 0)\) at the input \((z = 0)\) at time \(t = 0\). We immediately have the output of the waveguide equal to the impulse response

\[
p(\theta, z, t) = h(\theta, z, t),
\]

(44)

or to the transfer function in the frequency domain

\[
P(\Omega, z, \omega) = H(\Omega, z, \omega).
\]

(45)

In the case of the short pulse excitation at mode \(\theta_0\), i.e.,

\[
p(\theta, 0, t) = \delta(\theta - \theta_0)\delta(t),
\]

(46)

we obtain

\[
P(\Omega, z, \omega) = H(\Omega, z, \omega)e^{-i\Omega \theta_0}.
\]

(47)
Sometimes a very short pulse excites all the modes uniformly,

\[ p(\theta, 0, t) = \delta(t). \]  

(48)

Then, we have

\[ P(\Omega, z, \omega) = 2\pi \delta(\Omega) H(\Omega, z, \omega), \]

(49)

or in the case of the harmonic generation at the fundamental mode

\[ p(\theta, z, t) = \delta(\theta)e^{i\omega_0 t}, \]

(50)

one can find

\[ P(\Omega, z, \omega) = 2\pi \delta(\omega - \omega_0) H(\Omega, z, \omega), \]

(51)

and so on. Thus, on the basis of (43)-(45) it is possible to define the exact input conditions for the direct measurements of the impulse response and the transfer function of the system. Eqs. (46)-(51) enable us to compare different experimental data obtained under specific excitation conditions in various laboratories [8].

3 Linear system with memory

In this section, we will examine an one-channel linear transmission system with memory at time.

3.1 The power flow equation

Let us consider a linear causal system with memory in which the power flow equation has the form

\[ \frac{\partial s(z, t)}{\partial z} + \frac{1}{v_0} \frac{\partial s(z, t)}{\partial t} = -\alpha_0 s(z, t) + \int_{-\infty}^{t} q(t - \tau) s(z, \tau) d\tau - s(z, t) \int_{t}^{\infty} q(t - \tau) d\tau, \]

(52)

where \( s(z, t) \) is the signal power in position \( z \) at time \( t \) and \( q(\xi) \) is the memory function of the system (it is an even function). Let us solve this equation by means of the Laplace transform

\[ S(z, p) = \int_{0}^{\infty} s(z, t)e^{-pt} dt, \quad s(z, t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} S(z, p)e^{pt} dp. \]

(53)

Then, instead of (52), we have the equation

\[ \frac{\partial s(z, t)}{\partial z} + \frac{1}{v_0} \frac{\partial s(z, t)}{\partial t} = -\beta_0 s(z, t) + \int_{0}^{t} q(t - \tau) s(z, \tau) d\tau \]

(54)

together with the initial condition

\[ s(z, 0^-) = \lim_{t \to 0^+} s(z, t). \]

(55)
Of course, in (54)

\[ \beta_0 = \alpha_0 + \int_0^\infty q(t-\tau)d\tau = \alpha_0 + \int_{-\infty}^{t} q(t-\tau)d\tau. \]  

(56)

Using (53) we can rewrite (54) in the following way

\[ \frac{dS(z,p)}{dz} + k(p)S(z,p) = \frac{1}{v_0} s(z,0^+), \]  

(57)

where

\[ k(p) = \beta_0 + \frac{P}{v_0} - Q(p), \quad Q(p) = \int_0^\infty q(\xi)e^{-Qp\xi}d\xi. \]  

(58)

Eq. (57) describes the problem in the frequency domain \((p = i\omega)\) and has the solution

\[ S(z,p) = S(0,p)e^{-k(p)z} + \frac{e^{-k(p)z}}{v_0} \int_0^z s(\eta,0^+)e^{k(p)\eta}d\eta, \]  

(59)

where \(S(0,p)\) is the Laplace transform of the boundary condition \(s(0,t)\), i.e.,

\[ S(0,p) = \int_0^\infty s(0,t)e^{-pt}dt. \]  

(60)

Applying the inverse Laplace transform on (59), we obtain the solution of the problem in the time domain

\[ s(z,t) = \frac{e^{-\beta_0 z}}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{p(\frac{t}{v_0}+Q(p))} \int_0^\infty s(0,\tau)e^{-p\tau}d\tau dp + \frac{e^{-\beta_0 z}}{2\pi i v_0} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{p(\frac{t}{v_0}+Q(p))} \int_0^\infty s(\eta,0^+)e^{-[\beta_0 + \frac{P}{v_0} - Q(p)]\eta}d\eta dp. \]  

(61)

As it can be seen from the right-hand side of this equation, the solution consists of two terms. The first term depends on the boundary condition \(s(0,t)\), the second one on the initial condition \(s(z,0^+)\). Let us note that instead of the Laplace transform, the Fourier transform can be used for solution of the problem, but in that case, the influence of the initial condition \(s(0,t)\) cannot be included.

### 3.2 The transfer function and the impulse response

From (59), one can immediately get the transfer function of the system

\[ H(z,p) = \frac{S(z,p)}{S(0,p)} = e^{-k(p)z} + \frac{1}{v_0} \int_0^z s(\eta,0^+)e^{-k(p)(z-\eta)}d\eta. \]  

(62)

Then, by means of the inverse Laplace transform of (62), we can obtain the impulse response

\[ h(z,t) = e^{-\beta_0 z}q_c(z,t) + \frac{1}{v_0} \int_0^z s(\eta,0^+)q_c(z-\eta,t)e^{-\beta_0 (z-\eta)}d\eta, \]  

(63)
where \( q_e(z,t) \) is the inverse Laplace transform of \( \exp\{-[p/\nu_0 - Q(p)]z\} \), i.e.,

\[
q_e(z,t) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{-\left[p\nu_0 - Q(p)\right]z} e^{pt} \, dp.
\] (64)

Thus, the impulse response of the linear system with memory is given by (63). It contains two contributions. The first term on the right-hand side of this relation represents the impulse response when the initial condition is zero, the second term describes the response on the non-zero initial condition \( s(z,0^+) \). Often, the impulse response of the system is only determined in the case of the zero initial condition, or in the case when the response on the initial condition has vanished (because of the memory function behaviour).

### 4 Conclusions

As one can see [by comparison (2) with (54)] the above-described real multimode waveguide structure represents a general linear system with memory at space (i.e., at argument \( \theta \)) for which a principle of space causality is not necessary to take into account.

For real multimode waveguides, Set (1) can be derived from Maxwell’s equations for given boundary conditions by the method of the ideal waveguide modes or by the expansion series of so-called locally orthogonal modes [1]. However, on the basis of the law of power conservation, Set (1) can be written directly. The validity of this law is equivalent to the condition of symmetry of the coupling coefficients \( h_{mn} = h_{nm} \) in (1), or \( c(\phi, \theta) = c(\theta, \phi) \) in (2), or \( c(\phi - \theta) = c(\theta - \phi) \) in (30), i.e., functions \( c(\xi) \) in (32) and \( q(\xi) \) in (52) must be even. Set (1) and Eq. (2) describe the power flow in the multichannel system with no wave degeneration. But some multimode dielectric waveguides (e.g., multimode optical fibres) exhibit a mode degeneration, i.e., the single guided modes degenerate into mode groups with nearly the same values of propagation constants, attenuation coefficients, and propagation velocities. Then instead of (2), a similar integrodifferential equation can be written [4] and a transformed coupling function can be introduced. It can be shown that the power conservation is then equivalent to the symmetry of the transformed coupling function. Eq. (2) is of course, a special case of the integrodifferential equation describing the degenerative systems. Thus, the non-degenerative multichannel system can be taken as the degenerative one with the unit transform function. The integrodifferential equation which describes the degenerative structures can be solved in the same ways as (2).

There are also radiation and backguided modes in the multimode optical waveguides. To include the coupling among them and the guided modes we can either specify a new critical angle (\( \Theta \)) or express the additional losses of the guided modes caused by the interaction [4].

Eq. (2) describes the power propagation in the linear multichannel systems. However, it may be used also for the solution of some nonlinear propagation problems [9]. For example, in the case of optical Kerr effect, we can identify the nonlinear part of the refractive index with the waveguide perturbation, i.e., with the refractive index fluctuations of the waveguide medium which are responsible for the wave interactions.

The presented methods of the solution of the power flow equation (2) enable us to examine general linear multichannel transmission systems. In general, the problems that are described by a similar set of equations as (1) or directly by integrodifferential equation (2),
can be solved in the presented ways (e.g., multiple scattering [10], optical neural networks, etc.)

References


