Determination of Magnetic Induction and Current Density Values for Planar Cores to Operate with Minimal Magnetic Losses

Cláudio L. Ebert\textsuperscript{1}, Walter P. Carpes Jr.\textsuperscript{2}
GRUCAD - Dep. de Engenharia Elétrica - CTC – UFSC - Caixa Postal 476, Florianópolis, SC, CEP 88040-970
\textsuperscript{1}ebert@ifsc.edu.br, \textsuperscript{2}carpes@grucad.ufsc.br

João C. S. Fagundes
INEP - Dep. de Engenharia Elétrica - CTC - UFSC - Caixa Postal 5119, Florianópolis, SC, CEP 88040-970
fagundes@inep.ufsc.br

Abstract - This paper presents a methodology for determining the values of magnetic induction and current density in the design of transformers and inductors based on planar magnetic elements. From experimental values, datasheet parameters and through an optimization tool based on genetic algorithms, we obtain curves of magnetic induction and current density corresponding to operation with minimal magnetic losses. Hence, from the obtained results, it is possible to design a planar magnetic device operating with minimal losses.

Index Terms - magnetic losses, planar magnetic device.

I. INTRODUCTION

The planar magnetic elements are devices that have been recently used in static converters with great acceptance due to their better performance compared to standard magnetic elements [1]. Some advantages in using planar elements are listed below [8]-[11]:

- Structures based on planar elements have low geometric profile, which allow better volumetric efficiency and high power density in some applications;
- The increasing in area of the core central leg permits a reduction in the number of turns;
- With planar elements, it is possible to have low leakage inductance due to the simplicity in the interleaving of layers and small number of turns. This reduces the voltage spikes and oscillations that may destroy some types of semiconductor devices;
- They permit a decrease in the high-frequency winding losses, since the skin effect is reduced due to conductor thickness reduction;
- It is possible to obtain higher conversion efficiency due to the better magnetic coupling;
- We can have better thermal control, given that planar elements present a larger area/volume ratio, increasing the surface available for heat transfer, which reduces the thermal resistance;
- Easy construction with the use of Printed Circuit Board (PCB);
- Good repeatability, which is an indispensable characteristic in resonant topologies.
In addition to the advantages cited above, another factor that indicates planar magnetic elements to have a promising market is their geometric characteristic (Fig. 1): low profile is ideal for applications where minimum height is desired, e. g., power supplies for portable computers and for telecommunication systems.

![Fig. 1: (a) Conventional Magnetic Device; (b) planar magnetic device.](image)

Despite the presented advantages, very few results can be found in previous studies about these elements, making it difficult for industry to design devices taking advantage of planar structures [2].

The usual methodology for planar magnetic design generally focuses on the choice of magnetic induction \( B \) values in order to not saturate the magnetic core [3], disregarding magnetic losses. The adopted current density \( J \) values are often the same used in transformers and inductor constructed with conventional (cylindrical) conductors. Generally, the values chosen for \( B \) and \( J \) are constants and do not consider other parameters nor specifications of the magnetic element.

It is known that the magnetic losses are influenced by temperature [2]-[6]. In effect, there is a range of temperature in which the losses are minimized. Thus, this paper aims at presenting a methodology for obtaining the \( B \) and \( J \) values that make the core of magnetic elements operate at temperatures in which the magnetic losses are minimal.

The optimal values of magnetic induction and current density are determined through an optimization tool based on genetic algorithms. The required parameters were obtained from measurements taken on a magnetic device [6] specially built for this study. We developed some projects in order to validate the proposed method.

II. POWER LOSSES IN MAGNETIC ELEMENTS

Magnetic materials dissipate energy in the form of heat when subjected to time-varying magnetic fields. These losses are divided into "quasi-static" and dynamic losses. The quasi-static losses are caused by hysteresis phenomenon and do not depend on the excitation frequency. The dynamic losses are divided into losses caused by circulating currents due to alternating magnetic fields, and in anomalous losses. In this work, the total magnetic losses were obtained using an experimental set-up and a virtual instrumentation.

A. Workbench and virtual instrumentation for data acquisition

A full bridge inverter, shown in Fig. 2, has been implemented in order to carry out the measurements. This inverter allows control of voltage, frequency, duty cycle and dead time, making it possible to apply a symmetrical square waveform with zero average voltage.
In order to make the measurements more accurate and facilitate data processing, a virtual instrument was created with Labview 7.1® (software developed by National Instrument). The signals acquisition has been performed through a four channel oscilloscope connected to a computer, in which the virtual instrument has been programmed, as shown in Fig. 3. The information processing was made by the virtual instrument, presenting the results through graphics and tables.

By using a digital oscilloscope (Tektronix, DPO4104), we can simultaneously obtain the current, the voltages at two different points as well as the temperature in the core. The latter is measured by thermocouples connected to an amplifier, which sends the signal to one of the oscilloscope channels.

With the developed virtual instrument, it is possible to acquire the signals, process them and present the results through graphics (even in real time). The figures 4 and 5 show, respectively, one cycle of the voltage (in volts) in the winding 2 and one cycle of the current (in amperes) in the winding 1 as a function of time (in seconds) for no-load operation.
The $B$ (magnetic induction, in mT) and $H$ (magnetic field, in A/m) values can be obtained from the respective voltage and current. From these values, the $B$-$H$ curve can be plotted, as shown in Fig. 6.

![Fig. 6. B-H curve (B in mT and H in A/m)](image)

### B. Magnetic losses as a function of temperature

When the magnetic core operates at different temperatures, variations occur in the magnetic losses. This can be observed in Fig. 7, which shows the magnetic losses as a function of $B$ for three different temperatures.

![Fig. 7. Magnetic losses as a function of B for three different temperatures.](image)

Therefore, it is necessary to find a relation between magnetic losses and the operation temperature of the magnetic core. For that, we used the virtual instrument. It permits to visualize the evolution in time of both the core temperature and the magnetic losses, as shown in Fig. 8. We can clearly notice the magnetic loss variation with temperature.
Several measurements with four different cores, shown in Fig. 9, and using different parameters were carried out in order to verify the temperature influence on magnetic losses. The core characteristics are shown in Table I.

![Fig. 8](image1.png) Evolution of temperature (°C) and magnetic losses (W) with time (s).

![Fig. 9](image2.png) Transformers developed with planar cores E-PLT 64, E-PLT 38, E-PLT 22 e E-PLT 14 (from left to right, respectively).

**TABLE I**

<table>
<thead>
<tr>
<th>Core</th>
<th>V_e (mm³)</th>
<th>A_e (mm²)</th>
<th>l_e (mm)</th>
<th>mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-PLT 14</td>
<td>230</td>
<td>14.3</td>
<td>20.7</td>
<td>0.6</td>
</tr>
<tr>
<td>E-PLT 22</td>
<td>2040</td>
<td>78.5</td>
<td>26.1</td>
<td>6.5</td>
</tr>
<tr>
<td>E-PLT 38</td>
<td>8460</td>
<td>194</td>
<td>43.7</td>
<td>25</td>
</tr>
<tr>
<td>E-PLT 64</td>
<td>35500</td>
<td>511</td>
<td>69.7</td>
<td>100</td>
</tr>
</tbody>
</table>

Where:

- \( V_e \) – core volume;
- \( A_e \) – area of central leg;
- \( l_e \) – length of magnetic path.

For the parameters listed below, the obtained results are shown in Fig. 10.

Parameters corresponding to Fig. 10:

\[
B = 0.240 \, \text{T};
V = 45 \, \text{V};
\]
$f = 80$ kHz;
Waveform factor $K_v = 4$ (symmetrical square waveform);
$N = 3$ turns in a single layer;
Planar core E-PLT 38;
3F3 material (Ferroxcube).

For all the different parameters considered, we observed that the curves always follow the same tendency. Thus, we can apply a correction term to magnetic losses value in order to take into account the temperature dependence. The obtained curves show that the correction term, in the temperature range of $60 \degree C$ to $100 \degree C$, can be written as:

$$C(T_c) = ct - ct_1 T_c + ct_2 T_c^2,$$

where the coefficients must be determined from the experimental values by curve fitting.

For the 3F3 material, the coefficients are:
$ct = 3.95811$, $ct_1 = 0.07512 \times 10^{-2}$ °C$^{-1}$ e $ct_2 = 4.548 \times 10^{-4}$ C$^{-2}$.

B. Core temperature

The measured values of core temperature ($T_c$) versus magnetic losses ($P_m$) for Ferroxcube planar cores [4] are shown in Fig. 11.

Equations relating the magnetic losses ($P_m$, in watts) with the core temperature ($T_c$, in °C) for cores E-PLT22 (2), E-PLT38 (3) and E-PLT64 (4) can be obtained from the presented curves:
\[ T_c = -3.2216 P_m^2 + 57.126 P_m + T_a \]  \hspace{1cm} (2)

\[ T_c = -1.4468 P_m^2 + 30.441 P_m + T_a \]  \hspace{1cm} (3)

\[ T_c = -0.0416 P_m^2 + 16.289 P_m + T_a \]  \hspace{1cm} (4)

where \( T_a \) is the ambient temperature (in °C).

C. Winding temperature

The winding temperature \( (T_w) \) as a function of the current density \( (J, \text{ in A/mm}^2) \) was also measured, and the values are shown in Fig. 12.

The equations below, relating the winding temperature with the current density for cores E-PLT64 (5), E-PLT38 (6) and E-PLT22 (7), respectively, were obtained from the preceding curves:

\[ T_w = 0.0511 J^2 - 0.7813 J + T_a \]  \hspace{1cm} (5)

\[ T_w = 0.0537 J^2 - 0.3548 J + T_a \]  \hspace{1cm} (6)

\[ T_w = 0.0281 J^2 - 0.2984 J + T_a \]  \hspace{1cm} (7)

III. DETERMINATION OF MAGNETIC INDUCTION (B) AND CURRENT DENSITY (J)

The magnetic losses can be computed using:

\[ P_m = C_m V_e^{x} B^y f^z C(T_c) \]  \hspace{1cm} (8)

where \( V_e \) is the core volume (in mm\(^3\)); \( C_m, x, y \) and \( z \) are parameters depending on the material characteristics [6], \( f \) is the operation frequency (in kHz) and \( CT \) is a correction factor that depends on the core temperature \( T_c \) [5]. Fig. 10 shows \( P_m \) as a function of the core temperature. We observe that there is a temperature range in which the magnetic losses are minimal. Thus, the magnetic induction and current density values can be chosen in order to make the magnetic element operate at temperatures corresponding to minimal \( C(T_c) \).

A. Choice of B and J to planar magnetic devices with a single winding (inductor)

Single-winding magnetic elements are considered in this first analysis. From the equation of \( C(T_c) \), replacing the value of the \( T_c \) given in (3) gives:
The winding temperature is assumed as the ambient temperature for the magnetic core. In future analyses, this assumption will be reevaluated if the results are not satisfactory.

Thus, replacing the $T_w$ value in (9), we get the $C(T_c)$ equation in terms of $J$ and $P_m$ for the core with a single winding.

\[
C(T_c) = c_T - c_1 (-1.447 P_m^2 + 30.44 P_m + T_w) + \\
c_T (-1.447 P_m^2 + 30.44 P_m + T_c)^2.
\]  

Fig. 13 shows a graph of the $C(T_c)$ factor as a function of losses and current density. In this graph, we can observe a region where the value of $C(T_c)$ is minimal.

In order to find the points where the $C(T_c)$ value is minimal, we developed an optimization tool based on genetic algorithms [7]. As a result of the optimization process, we found the values of $P_m$ and $J$ corresponding to minimal $C(T_c)$.

Table II shows the parameters of the real genetic algorithm developed in this work.

<table>
<thead>
<tr>
<th>GENETIC ALGORITHM PARAMETERS</th>
<th>10 individuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>2</td>
</tr>
<tr>
<td>Number of variables per individual</td>
<td>100%</td>
</tr>
<tr>
<td>Crossover probability</td>
<td>100%</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>20%</td>
</tr>
<tr>
<td>Maximum number of generations</td>
<td>100</td>
</tr>
</tbody>
</table>

Depending on the initial values of the current density and magnetic losses, the optimization algorithm converges to different points, showing that there are several points at which $C(T_c)$ is minimal.

Therefore, it was necessary to implement a new optimization algorithm that, from a set of initial values ($P_m, J$), can generate a set of values ($P_m, J$) minimizing $C(T_c)$. With the values of ($P_m, J$), we can plot the graph shown in Fig. 14.
By curve fitting, it is possible to obtain an equation that permits to calculate the magnetic losses from the current density in the region where $C(T_c)$ is minimal:

$$P_m = -0.0019J^2 + 0.0093J + 2.1205$$  \hspace{1cm} (11)

From (6), $B$ is given by:

$$B = (P_m / (C_m Ve f^x C(T_c)))^{1/y}$$  \hspace{1cm} (12)

By replacing $P_m$ and $C(T_c)$ by their corresponding values, we obtain an expression linking the current density and the magnetic induction for the planar element to operate at temperatures of minimal magnetic losses:

$$B = ((-0.0019J^2 + 0.0093J + 2.1205) / (C_m Ve f^x (ct - ct_1 (-1.447(-0.0019J^2 + 0.0093J + 2.1205)^2 + 30.44(-0.0019J^2 + 0.0093J + 2.1205) + (0.0537J^2 - 0.3548J + T_a))^2 + ct_2 (-1.447(-0.0019J^2 + 0.0093J + 2.1205)^2 + 30.44(-0.0019J^2 + 0.0093J + 2.1205) + (0.0537J^2 - 0.3548J + T_a))^2))^{1/y}.$$  \hspace{1cm} (13)

Finally, from (13), one can plot a graph of the magnetic induction as a function of the current density for core PLT E-38, material 3F3, operating at 40, 60, 80, 100 and 120 kHz, as is shown in Fig. 15.

By curve fitting, it is possible to obtain an equation that permits to calculate the magnetic losses from the current density in the region where $C(T_c)$ is minimal:

$$P_m = -0.0019J^2 + 0.0093J + 2.1205$$  \hspace{1cm} (11)

From (6), $B$ is given by:

$$B = (P_m / (C_m Ve f^x C(T_c)))^{1/y}$$  \hspace{1cm} (12)

By replacing $P_m$ and $C(T_c)$ by their corresponding values, we obtain an expression linking the current density and the magnetic induction for the planar element to operate at temperatures of minimal magnetic losses:

$$B = ((-0.0019J^2 + 0.0093J + 2.1205) / (C_m Ve f^x (ct - ct_1 (-1.447(-0.0019J^2 + 0.0093J + 2.1205)^2 + 30.44(-0.0019J^2 + 0.0093J + 2.1205) + (0.0537J^2 - 0.3548J + T_a))^2 + ct_2 (-1.447(-0.0019J^2 + 0.0093J + 2.1205)^2 + 30.44(-0.0019J^2 + 0.0093J + 2.1205) + (0.0537J^2 - 0.3548J + T_a))^2))^{1/y}.$$  \hspace{1cm} (13)

Finally, from (13), one can plot a graph of the magnetic induction as a function of the current density for core PLT E-38, material 3F3, operating at 40, 60, 80, 100 and 120 kHz, as is shown in Fig. 15.

### B. Choice of $B$ and $J$ for planar magnetic devices with two windings (transformer)

The relationship between $B$ and $J$ depends on the core temperature and winding temperature, in addition to the core specifications and the design parameters [8]-[11].
For two-winding planar elements, it is necessary to consider the temperature influence on the two windings. Thus, the core temperature depends on both the magnetic losses and the temperature of the two windings.

Therefore, from (3) we have:

\[ T_c = -1.447P_m^2 + 30.44P_m + T_{w2}, \]

(14)

where \( T_{w2} \) is the temperature in the winding 2. To simplify the analysis, we consider that the winding temperature and the core temperature are equal.

The temperature in the winding 2 as function of the current density is calculated using:

\[ T_{w2} = 0.0537J_2^2 - 0.3548J_2 + T_{w1}. \]

(15)

Hence, the ambient temperature for the winding 2 and the temperature of winding 1 are assumed to be the same.

The temperature in winding 1 as function of the current density is calculated from (6). Thus, \( C(T_c) \) is given by:

\[ C(T_c) = \frac{1}{2} \left[ P_m + \left( 0.0537J_1^2 - 0.3548J_1 + T_{w1} \right) \right] . \]

(16)

To determine the values of \( P_m, J_1 \) and \( J_2 \) that optimize (minimize) this function, we used the developed genetic algorithm tool.

The values of \( J_1 \) and \( J_2 \) are assumed to be the same \( (J_1 = J_2 = J) \). Thus, we get a relationship between magnetic loss and current density that can be used for both primary and secondary winding.

The curve with the values of \( P_m \) and \( J \) for a two-winding magnetic element is shown in Fig. 16.

![Fig. 16. Magnetic losses as a function of current density considering \( J_1 = J_2 \).](image)

From the Fig. 16 we can obtain:

\[ P_m = -0.0036J^2 + 0.0159J + 2.1462. \]

(17)

From (12) and by using \( C(T_c) \) and \( P_m \) given in (16) and (17), respectively, we obtain an expression relating the current density and the magnetic induction corresponding to operation at a temperature of minimal magnetic losses.

Fig. 17 shows current density as a function of magnetic induction for five different frequencies.
Fig. 17 Magnetic induction as a function of current density for the two-winding magnetic element.

Figs. 18, 19 and 20 show the curves for E-PLT22, E-PLT38 and E-PLT64 cores, considering one and two windings.

Fig. 18. Magnetic induction as a function of current density for E22 (3F3) core with one and two windings.

Fig. 19. Magnetic induction as function of the current density for E38 (3F3) core with one and two windings.
IV. RESULTS

From the values presented in Figs. 18, 19 and 20, we have designed three transformers used in full bridge inverters.

The specifications and the obtained results are shown in Table III.

**TABLE III. SPECIFIED, CALCULATED AND EXPERIMENTAL VALUES, TO TRANSFORMER DESIGN**

<table>
<thead>
<tr>
<th>Design n°:</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specified values</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency (kHz)</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>Primary voltage (V)</td>
<td>50</td>
<td>26</td>
<td>18</td>
</tr>
<tr>
<td>Secondary voltage (V)</td>
<td>50</td>
<td>26</td>
<td>18</td>
</tr>
<tr>
<td>Secondary current (A)</td>
<td>2.06</td>
<td>4.3</td>
<td>2.3</td>
</tr>
<tr>
<td><strong>Calculated values</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core</td>
<td>E-PLT38</td>
<td>E-PLT38</td>
<td>E-PLT22</td>
</tr>
<tr>
<td>Current density (A/mm²)</td>
<td>15.14</td>
<td>13.29</td>
<td>28.90</td>
</tr>
<tr>
<td>Magnetic induction (T)</td>
<td>0.201</td>
<td>0.209</td>
<td>0.238</td>
</tr>
<tr>
<td>Number of turns in the winding 1</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Number of turns in the winding 2</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Width of the winding 1 (mm)</td>
<td>1.94</td>
<td>4.62</td>
<td>1.13</td>
</tr>
<tr>
<td>Width of the winding 2 (mm)</td>
<td>1.94</td>
<td>4.62</td>
<td>1.13</td>
</tr>
<tr>
<td>Magnetic losses (W)</td>
<td>1.56</td>
<td>1.72</td>
<td>0.5</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>82</td>
<td>82</td>
<td>82</td>
</tr>
<tr>
<td><strong>Experimental values</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary voltage (V)</td>
<td>50.92</td>
<td>26.4</td>
<td>18.6</td>
</tr>
<tr>
<td>Secondary voltage (V)</td>
<td>49.47</td>
<td>25.5</td>
<td>17.6</td>
</tr>
<tr>
<td>Secondary current (A)</td>
<td>2.08</td>
<td>4.4</td>
<td>2.3</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>84</td>
<td>84</td>
<td>87</td>
</tr>
<tr>
<td>Error in the temperature (%)</td>
<td>2.4</td>
<td>2.3</td>
<td>5.7</td>
</tr>
</tbody>
</table>

The planar transformers were designed to observe the core temperature operating with the obtained values of magnetic induction and current density.

It was observed that there is a small difference between calculated and experimental values. The largest difference was for the design number 3, where the designed winding width was 1.1 mm, but it was actually implemented as 1 mm. This reduction of 10% in the winding width causes an increase in the current density and, consequently, the measured temperature was higher than the theoretical value.
CONCLUSIONS

This paper presented a procedure for planar magnetic elements design aiming operation with minimal magnetic losses. From the equations and the results of measurements presented here, it is possible to choose the magnetic induction and the current density values in order to minimize the magnetic losses.

The projects developed with the procedure presented in this work considered planar cores of different volumes, operating at different frequencies. The measured temperatures were close to the theoretical values for all projects.

Finally, it is important to remark that, in this work, we only considered planar cores made with 3F3 material. But the same procedure can be applied to other cores, of different materials and sizes, in order to build a larger database for the design program [2].

REFERENCES