Non-Linear Surface Waves in a Left Handed-Magnetized Ferrite Structure

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Abstract— A theoretical investigation of nonlinear surface waves propagating in a planar waveguide structure of a nonlinear Left-handed material (LHM) cover and magnetized Ferrite substrate, has been analyzed. The dispersion relation and the effect of intensity of electric field on the propagation characteristics have been examined. It is found that both of the nonlinearity and the magnetization of Ferrite change the direction of the dispersion curves.

Index Terms— Dispersion relation, Left-handed materials, Power flow, Nonlinear waves.

I. INTRODUCTION

Novel theoretical and experimental \cite{1-4} studies have shown the possibility to create novel types of microstructured materials which demonstrate the property of negative refraction. Properties of such materials were analyzed theoretically by Veselago long time ago \cite{5}. In particular, the composite materials created by arrays of wires and split-ring resonators(SRR) were shown to possess the negative real part of the magnetic permeability and dielectric permittivity for microwaves. These materials are often referred as left-handed material (LHM).

In general, the properties of left-handed materials were studied in the linear regime of wave propagation when both magnetic permeability and dielectric permittivity of the material are assumed to be independent on the intensity of the electromagnetic field. However, the study of nonlinear properties of left-handed materials have attracted a little attention \cite{6,9}. Nonlinear response of such a composite can be characterized by two different contributions. The first one is the dielectric permittivity depends on the intensity of electric field $E$. The second contribution into the nonlinear properties of the composite material comes from the lattice of SRR, which depends on the magnetic field $H$. In this paper, we analyze a nonlinear surface waves in a planar waveguide containing a nonlinear left-handed materials with a magnetized ferrite. The interest in studying of a magnetized ferrite that its permeability is described by tensor, which has the ability to change the direction of...
electromagnetic surface waves. Such a phenomena can be used in optoelectronic circuits. Also, we study, the case were the permeability of left-handed materials is equal -1. This material can be considered as non-magnetized LHM.

II. NONLINEAR DISPERSION RELATION

The existence of linear surface waves can be achieved under a special condition, at a planar interface between a semi-infinite of two different dielectric media. In particular, the existence of TM surface waves requires that the dielectric constants of two materials separated by an interface have different signs, whilst for TE surface waves the magnetic permeability of the materials should be of different signs. Materials with negative $\varepsilon$ are readily exist,(c.g., metal), whilst materials with negative $\mu$ is not exist in nature. This explain why only TM surface waves have been interested .A model structure that produces TE surface waves at a single boundary can be demonstrated theoretically [10]. The necessary condition of continuity of both the electric field and its gradient is satisfied if one of the semi-infinite media displays a positive Kerr-type nonlinearity.

More recently [10], it has shown, theoretically, that if one of the semi-infinite media is a left-handed medium then the boundary conditions for TE surface waves can exist.

In this paper, we study nonlinear waves propagating along interface between linear right handed (RH) magnetized ferrite and nonlinear left handed cover (LHM) as shown in fig.1.

The elements of the permeability tensor (bias field in the $z$ direction)[10],

$$
\begin{bmatrix}
\mu_1 & -j\mu_2 & 0 \\
 j\mu_2 & \mu_1 & 0 \\
 0 & 0 & \mu_z
\end{bmatrix}
$$
The description of the elements of the permeability tensor depends on the range of the bias fields. Hansson and Filipsson[11] had given the formulas for the whole range of bias fields, with the derivations of the tensor elements being continuous:

$$\mu_1 = \mu_e + (1 - \mu_e) m^{3/2} + \frac{h_i}{h_i^2 - w^2}, \mu_2 = \frac{m w}{h_i^2 - w^2},$$

with the normalized values

$$h_i = \frac{H_i}{M_s}, \ m = \frac{M}{M_s}, \ w = \frac{\omega \ v_m}{\omega_m}, \ \omega_m = \gamma \mu_o M_s, \ \mu_e = \frac{1}{3} \left(1 + 2 \sqrt{1 - \frac{1}{w^2}}\right).$$

$M_s$ is the saturation magnetization, $M$ the actual magnetization, $H_i$ is the internal magnetic bias field, and $\gamma$ the gyromagnetic ratio. Here, the value of $m = \frac{M}{M_s}$ should be greater than zero to consider the ferrite medium is magnetized.

The propagation of monochromatic waves with frequency $\omega$ is governed by the scalar wave equation, which for the case of the TE waves is written for $z$-component of the electric field,

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c^2} \varepsilon \mu - \frac{1}{\mu} \frac{\partial \mu}{\partial x} \frac{\partial}{\partial x}\right] E_z = 0,$$

where $\mu$, $\varepsilon$ are magnetic permeability and dielectric permittivity, respectively; $\omega$ is the angular frequency; and $c$ is the speed of light in vacuum.

For the linear medium (Magnetized Ferrite), the equation (1) can be written the form:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c^2} \varepsilon_f \mu_v\right] E_z = 0,$$

where $\varepsilon_f$ is the dielectric permittivity, and $\mu_v = \frac{\mu_2^2 - \mu_1^2}{\mu_1}$. Solution of equation (2) has the form

$$E_z(x, y) = E_o \exp(k_1 y) \exp(ikx),$$

where

$$k_1 = \left[k^2 - \varepsilon_f \mu_v \left(\frac{\omega}{c}\right)^2\right]^{1/2}$$

is the transverse wave number which characterize the inverse decay length of the nonlinear surface wave in the corresponding medium, $E_o$ is the wave amplitude which is determined from the boundary condition, and $k$ is a wave vector.

The nonlinear (LHM) cover is Keer-type medium and has a dielectric permittivity function [7],

$$\varepsilon_{eff}^{NL} = \varepsilon_{eff} + \alpha |E|^2,$$
where the first term represents the linear property of dielectric function. For RH dielectric medium, positive $\alpha$ corresponds to self-focusing nonlinear material, whilst negative $\alpha$ characterizes defocusing effects. However, this classification is reversed in the case of LHM, and, for example, the self-focusing nonlinearity corresponds to negative $\alpha$ since $\epsilon$ is negative for LH materials.

By taking into account relation (4), we rewrite equation (2) for the case of TE-polarized wave in nonlinear (LHM) cover as follows:

$$
\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c^2} (\epsilon_{\text{eff}}(\omega) \mu_{\text{eff}}(\omega) + \mu_{\text{eff}}(\omega) \alpha |E|^2) \right] E_z = 0 ,
$$

(5)

where $\epsilon_{\text{eff}}$ is selected in the form of the commonly used function for plasmon investigations [7], and $\mu_{\text{eff}}$ is constructed in an analogous form, i.e.

$$
\epsilon_{\text{eff}}(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \mu_{\text{eff}}(\omega) = 1 - \frac{F \omega^2}{\omega^2 - \omega_r^2},
$$

(6)

the losses are neglected. Here $\omega_p$ is an effective plasma frequency that depends upon the geometry of the system, $\omega_r$ is a resonance frequency and $F$ is a parameter that also depends upon the system structure. One of the important physical consequence of the negative of both $\mu_{\text{eff}}(\omega)$ and $\epsilon_{\text{eff}}(\omega)$ is the appearance of the back waves [6] in the material. This critical result follows from a plane wave treatment of Maxwell's equations and the Poynting vector $S$ which is

$$
S = \frac{1}{2} \left( \frac{1}{\omega \mu_{\alpha} \mu_{\text{eff}}(\omega)} \right) |E|^2 = \frac{1}{2} \left( \frac{1}{\omega \epsilon_{\alpha} \epsilon_{\text{eff}}(\omega)} \right) |H|^2
$$

(7)

It is immediately apparent that $k$ is anti-parallel to $S$, under the condition $\mu_{\text{eff}}(\omega) < 0$ and $\epsilon_{\text{eff}}(\omega) < 0$, and that backward waves can be expected.

The solution of equation (5) in the form $E_z(y) = \Psi_z(y) \exp(ikx)$, where $\Psi(x,y)$ is found in the standard form

$$
\Psi_z(y) = \frac{ck_2}{\omega} \sqrt{\frac{2}{\alpha \mu_{\text{eff}}(\omega)}} \text{sech} \left( k_2 (y - y_o) \right),
$$

(8)

where $k_2 = \left[ k^2 - \epsilon_{\text{eff}}(\omega) \mu_{\text{eff}}(\omega) \left( \frac{\omega}{c} \right)^2 \right]^{1/2}$, and $y_o$ is the center of the sech-function which should be determined from matching conditions of the continuity of the tangential components of the electric and magnetic fields at the interface.

By applying the boundary condition, the nonlinear dispersion relation of the surface waves is found in the form
\[
\frac{k_2 \mu_0 v}{\mu_{\text{eff}} (\omega)} = k_1 - \frac{\mu_2}{\mu_1} k ,
\]

where \( v = \tanh(k_2 y_0) \).

III. NUMERICAL CALCULATION

Using equation (6) and the dispersion relation (9), TE surface waves can be generated with a dispersion curves having a negative gradient, indicating a negative group velocity. This negative group velocity depends critically on the parameter \( m = \frac{M}{M_s} \) of the magnetized ferrite, as shown in fig. (2).

In the magnetized ferrite (RHM) substrate the group and phase velocities are in the same direction (forward traveling waves). In the (LHM) cover the group and phase velocities are in opposite direction (backward traveling waves). In the planar waveguide there is a competition between the forward and backward traveling waves, and the total power flow will be either forwards or backwards, depending on the sum of the positive flowing power in the substrate and negative flowing power in the cover. This gives a characteristic maximum in the dispersion relation.

Fig. (3) shows that the position of the dispersion curves at which this maximum occurs is determined by the cover permittivity. Since the cover is a nonlinear medium, the permittivity depends on the intensity of the electric field, the position of the maximum in the dispersion curve can be controlled by the intensity of the electric field. Since the maximum in the dispersion curve indicates the value of the wave constant at which the total power flow changes from positive to negative, then there is possibility that the direction of the power can be controlled by the intensity of the electric field. Fig. (3) also shows the shift in the dispersion curves due to the change of the nonlinearity, i.e. the change of the intensity.
The power flow in the substrate and cover given by:

\[ P_s = \frac{k^2}{\omega \mu_0 \alpha \mu_{eff}^2 (\omega) k_o^2} (1 + v), \]  

(10-a)

\[ P_c = \frac{1}{2} \left[ \frac{k + \mu_2 (k - \frac{\mu_2}{\mu_1} k_1) k_2^2}{\omega \mu_0 \alpha \mu_{eff} \mu_k k_o^2} \right] \left( 1 - v^2 \right), \]  

(10-b)

Fig. 2. Shows the effective wave index \( \beta = \frac{k}{k_o} \) versus frequency \( f \) for different values of \( m \): curve 1, \( m = 0.2 \); curve 2, \( m = 0.4 \); curve 3, \( m = 0.6 \); curve 4, \( m = 0.8 \). The interface nonlinearity, \( \nu = 0.2 \), \( h = 0.3 \), \( \mu_0 M_s = 0.177 T \) and \( \gamma = 1.7 \times 10^{11} (T_s)^{-1} \).

Fig. 3. Shows the effective wave index \( \beta = \frac{k}{k_o} \) versus frequency \( f \) for different values of nonlinearity \( \nu \): curve 1, \( \nu = 0.1 \); curve 2, \( \nu = 0.15 \); curve 3, \( \nu = 0.2 \); curve 4, \( \nu = 0.25 \), and \( m = 0.6 \).
Fig. (4) shows the corresponding positions of the power flow change from positive to negative. In fig. (5), we notice that the change of the direction of total power flow depending on the magnetization \((m)\) of ferrite.

Fig. 4. The normalized power \((P/P_o)\) versus the effective wave index for different values of the frequency \((f)\): curve 1, \(f = 5.1\) GHz; curve 2, \(f = 5.15\) GHz; curve 3, \(f = 5.2\) GHz, \(P_o = \frac{1}{2\mu_o\omega^2}\) and \(m = 0.3\).

Fig. 5. The normalized power \((P/P_o)\) versus the effective wave index for different values of \(m\): curve 1, \(m = 0.25\); curve 2, \(m = 0.3\); curve 3, \(m = 0.35\); curve 1, \(m = 0.4\) and \(f = 5.1\) GHz.
In the non-magnetized RH material, the permeability $\mu = 1$. So, if we suggest a new type of a nonlinear nonmagnetic LHM, where the permeability is $\mu = -1$, we notice that the dispersion curves having only positive gradient, indicating a positive group and phase velocities, as shown in fig. (6). In fig. (7), we notice that the power flow has only a positive value, i.e., the power can not change its direction due to the change of the intensity.
V. CONCLUSION

The dispersion characteristics of nonlinear surface waves propagating in a planar waveguide of nonlinear LHM cover and magnetized Ferrite substrate have been investigated theoretically. We have found that the direction of the dispersion curves can be controlled by the magnetization of Ferrite, and the intensity of the electric field. In the case of nonlinear nonmagnetic LHM, we have seen that, the direction of the power flow can not be controlled by the intensity of the electric field.

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