Multiresolution Source Method for the Analysis of Planar Microstrip Structures

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Abstract

This paper presents an integral equation analysis of microstrip circuits and antennas. This approach uses an excitation on the plane of the circuit in conjunction with compactly-supported wavelets trial functions. This Multiresolution Source Method (MRSM) generates a sparse linear system. The application of the Discrete Wavelet Transform (DWT) allows a significant reduction of the central processing unit time and the memory storage. Results for a step discontinuity, integrated filters and patch antennas are presented to show the efficiency of the method.

I. Introduction

To deal with electromagnetic scattering problems, solving integral equation by using source method is efficient [1]. The source is described by an arbitrary function. Galerkin’s method is used to model the current density on the circuit. In Galerkin’s method, the use of appropriate trial functions for representing the unknown current distribution is important for the accuracy and efficiency of the solution. The rooftop expansion functions approach is mostly developed for complex structures. Therefore, this approach leads to a large and dense matrix [2]. To increase the efficiency of such methods, the use of wavelet bases in multiresolution approaches is often proposed [3,4].

This work presents an innovative multiresolution source method for analysing microstrip circuits and antennas. Different bases of compactly-supported wavelets are directly used as trial functions. Biorthogonal spline and Daubechies wavelets are used to characterize microstrip circuits.

Since the presented approach is an analysis of planar circuits in a completely enclosing, shielding, rectangular box [5], the analysis of radiating structures must be done, as presented in sonnet software manual, under certain conditions. In this paper, patch antennas are studied. Daubechies wavelets are used to model the current travelling on the feed line toward the patch edge, and Biorthogonal Spline wavelets are applied on the patch.

The application of the DWT allows the reduction of the computation time [3,4]. It is also shown that the use of wavelets bases produces a matrix that localized, and with a good condition number. These are both necessary conditions to allow an efficient sparsification of the matrix, which permits a significant reduction of the central processing unit time and the memory storage.
II. Theoretical developments

II.1 Source Method: Case of a one-port planar microwave circuit.

To explain clearly the development of the theoretical formulation, we use to a large extent the formalism presented in [1]. The structure is a shielded metallic box (Fig. 1). The planar circuit is supposed lossless. A voltage source placed on the circuit plane is used and the characterization is made by modelling the current density with the help of appropriate trial functions.

On the plane of the circuit, the electric tangential field is related to the total current density by the impedance operator \( \hat{Z} \):

\[
\vec{E}_T = \hat{Z} \vec{J}
\]  

(1)

\( \hat{Z} \) is a projection operator on the \( f^{TE}_{mn} \) and \( f^{TM}_{mn} \) modes of the empty waveguide.

The solution of the problem consists in satisfying the following boundary conditions:

\[
\vec{E}_T = 0 \quad \text{on the metal} \quad (2.a)
\]

\[
\vec{J} = 0 \quad \text{on the insulator} \quad (2.b)
\]

Fig. 1. A shielded two-ports planar microwave circuit.

Using one source formulation, an arbitrary excitation \( \vec{E}_0 \) is defined on the metal subregion.

On the plane of the strip, the current density on the metal subregion \( \vec{J}_M \) is expressed in term of trial functions basis ( \( \vec{G}_i \), \( i = 1 \) to \( n_s \)).

The planar dipole can be modelled with equivalent diagram as shown in Fig. 2:
\[ \hat{Y}_1 \text{ and } \hat{Y}_2 \text{ are the admittance operator: } \hat{Z} = (\hat{Y}_1 + \hat{Y}_2)^{-1} \]

\[ \vec{E}_0 \text{ defined on } (M), \vec{E}_0 = V_0 \vec{e}_0, \vec{e}_0 \text{ is the source function of amplitude } V_0. \]

\[ \vec{J}_M \text{ defined on } (M), \vec{J}_M = \sum_{i=1}^{ns} X_i \vec{G}_i \]

Next, using Galerkin’s procedure, (1) is transformed into the following set of linear equations:

\[ \sum_{j} \langle \vec{G}_i, \hat{Z} \vec{G}_j \rangle X_j = \langle \vec{G}_i, \vec{G}_0 \rangle \quad \forall i \quad (3) \]

Thus, we obtain a matrix equation: \[ \mathbf{AX} = \mathbf{B} \quad (4) \]

We define the admittance, seen by the source as follows: \[ \text{Yin} = \langle \vec{J}_M, \vec{E}_0 \rangle = \langle \sum_{i=1}^{ns} X_i \vec{G}_i, \vec{E}_0 \rangle \]

**II.2 Wavelets and Scaling Functions:**

Let \( \phi(x) \) and \( \psi(x) \) be the scaling function and the mother wavelet associated with a multiresolution decomposition of \( L_2(\mathbb{R}) \) [6,7]. The trial functions basis \( \{ \phi_{jn}, \psi_{jn} \} \) are dilated and translated versions of the mother wavelet and the scaling function. \( \phi_{jn}(x) \) is defined via \( \phi(x) \) as follows: \( \phi_{jn}(x) = 2^{j/2n} \phi(2^j x - n) \). Where \( n \) is the translation factor, and \( j \) is the resolution level. A similar definition holds for \( \psi_{jn}(x) \). The spectrum of \( \phi_{jn}(x) \) is centred in the low-frequency regime, while that of \( \psi_{jn}(x) \) is centred in the bandpass regime. An approximation of the function \( J(x) \) at a resolution \( k \) can be written as the sum of two mutually orthogonal functions, namely, smooth \( (J^S, \text{ macro scale}) \) and detail \( (J^d, \text{ micro scale}) \) components. We have \( J(x) = J^S(x) + J^d(x) \), where:

\[ J^S(x) = P_J J(x) = \sum_n s_n \phi_{jn}(x), \quad s_n = \langle J, \phi_{jn} \rangle \]
\[ J^d(x) = D^k_j J(x) = \sum_{m=j}^{k-1} \sum_{n} d_{mn} \psi_{mn}(x), \quad d_{mn} = \langle J, \psi_{mn} \rangle \]

Here, \( \langle \cdot, \cdot \rangle \) denotes the inner product of \( L_2(R) \) and \( j \langle k \rangle \) is the reference smoothing resolution.

A function \( \psi(x) \) is said to have a vanishing moment of order \( N \) if:

\[
\int_{-\infty}^{\infty} x^n \psi(x) \, dx = 0 \quad \forall n = 0, 1, \ldots, (N-1)
\]

In practice, a wavelet with \( N \) vanishing moments enables the cancellation of all wavelet coefficients of a polynomial signal whose degree is less than \( N \). Thus if \( f \) is polynomial signal of degree less than \( N \) on the support of \( \psi_{j,n} \), then \( c_{j,n}(f) = \langle f, \psi_{j,n} \rangle = 0 \) [8].

This result is quite significant because it enables high compression rates (many wavelet coefficients are zero or negligible). If the \( f^{TE}_{mn} \) and \( f^{TM}_{mn} \) modes of the empty waveguide are smooth enough to be approximated by a polynomial expression of order less than \( N \), then (3) clearly shows that the associated matrix element vanishes or becomes very small.

A thresholding procedure is applied to set to zero all elements in the matrix whose magnitudes fall below the threshold. This threshold is adjusted to get an error of two percent or less in the current density [2]. A compression rate is defined as follows:

\[
R(\%) = \frac{\text{NbZero}}{\text{ns}^2} \times 100
\]

Where \( \text{NbZero} \) is the number of elements set to zero in the compression matrix \( (A) \) and \( \text{ns} \) is the size of the linear system.

**II.3 Choice of sources and trial functions**

The modelling of a planar circuit consists of determining its behaviour by calculation and programming. Efficiency and accuracy of the computation can be improved by the choice of appropriate sources and trial functions.

In our work different kinds of sources and compactly-supported wavelet bases have been tested. In this paper, a constant source has been chosen, composed of only one component in the \( z \) direction. This source function permits a good numerical convergence. Trial functions \( \{G_i\} \) are two-dimensional wavelet bases. So, \( G_i \) is a scaling function \( \phi_{jn} \) or a wavelet function \( \psi_{jn} \). Biorthogonal spline and Daubechies bases have been chosen for modelling microstrip circuits and antennas. The Discrete Wavelet Transform (DWT) is also exploited for the computation of inner products.
III. Application to microstrip circuits

In order to assess its efficiency, the Multiresolution Source Method is developed for analysing discontinuities in microstrip lines and integrated filters.

A- Step discontinuity

We first consider a step discontinuity in a microstrip line (Fig. 1). The structure is a planar circuit, composed of two lines with different widths w₁ and w₂, laid on a dielectric substrate. Circuits are assumed to be lossless with infinite thin metallization and isotropic substrate. The dimensions of the structure are: a=12.7mm, b=12.7mm, x₁= w₁=1.27mm, w₂=5.08 mm and εᵣ=10.

Scattering parameters are calculated for frequencies between 2-10 GHz. The Multiresolution Source Method analysis of the step discontinuity demonstrates the efficiency of the proposed technique compared to the conventional source method approach using rooftop trial functions.

The determination of discontinuity’s characteristics requires a convergence study. Tab. 1 shows that, for a frequency of 8 GHz, the scattering parameters converge from 3800 modes TE and TM and 192 rooftop Trial Functions (T.F.). Using Biorthogonal Spline (bior 6-8) wavelets, only 2000 modes TE and TM and 98 trial functions are needed to obtain the convergence of results.

| Trial functions | Number of Modes | R, % | Memory, Kb
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<tr>
<td></td>
<td>T.F.</td>
<td>TE</td>
<td>TM</td>
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<td>Roof Top</td>
<td>192</td>
<td>3800</td>
<td>40</td>
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<tr>
<td>Biorthogonal Spline</td>
<td>98</td>
<td>2000</td>
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Fig. 3 shows a grayscale plot obtained by taking the elements of the matrix A. This clearly demonstrates that the nonnegligible elements are located on or near the diagonal, which correspond to self-interactions or near-functions interactions. However, the use of wavelets trial functions allows a considerable reduction of nonnegligible elements.

Furthermore, as shown by table 1, for an error of 2 % or less in the current density, the compression rate is 40 % with rooftop functions and 80% with wavelets trial functions.
Thus, the use of source method with wavelets expansion decreases the computation time by a factor of five and the required memory by a factor of ten.

The effect of compression rate on the accuracy of the solution is considered. Fig. 4 reports the frequency response of $|S_{12}|$ of the considered structure for different levels of compression rate. The results obtained with the full matrix (R=0%) are in perfect agreement with the scattering parameters given by the reference [1]. Moreover, it appears that with a sparsification rate of 50%, results are essentially indistinguishable from those of the reference, and with a sparsification of 80% an error of 2% is obtained.

In order to further prove the good performances of the presented method, we consider the application of the Multiresolution Source Method for modelling low-pass and band-pass filters. The results are compared with those obtained using a published Finite-Difference Time-Domain method [9].

**B- Low-pass filter**

We first consider a low pass filter depicted in Fig. 5. The scattering parameters are calculated for frequencies between 2-12 GHz.
Biorthogonal spline (bior 1-5) wavelets are used as trial functions. The convergence of results requires 2000 modes TE and TM and 112 trial functions. The scattering parameters are shown in fig. 6. The results obtained with a sparsity of 70% are in perfect agreement with those published by Shao and Hong [9].

C- Band-pass filter

A band-pass filter is shown in Fig. 7. The scattering parameters are calculated for frequencies between 2-10 GHz.
Daubechies (db5) wavelets are used as trial functions. The convergence of results requires 2000 modes TE and TM, and 108 trial functions. The scattering parameters are shown in fig. 8. The results obtained with a compression rate of 76% are in good agreement with those given by the reference [9].

As presented, the results obtained for the low-pass filter and the band-pass filter are in good agreement with those obtained using a published Finite-Difference Time-Domain method [9]. However, we can observe that the magnitude of $S_{11}$ presented by [9] is always smaller than the results obtained in this analysis. The difference is due to the shielding effects. In fact, our approach is a shielding structure analysis whereas the method presented in [9] is an open structure analysis. Thus, the difference between the curves increases at high frequencies because of the box mode propagation.
IV. Application to radiating structures

In this study, we consider the application of a Multiresolution Source Method for the analysis of an edge-fed rectangular patch and an inset fed patch antennas.

Since the presented approach is an analysis of planar circuits in a completely enclosing, shielding, rectangular box, the analysis of radiating structures must be done under certain conditions:
- make both of the lateral substrate dimensions greater than one wavelength,
- make sure that the sidewalls are far enough from the radiating structure (one wavelength),
- place the top cover outside the fringing fields (half wavelength) and set a 377Ohms/square top cover to approximate removing the top cover of the box.

We first consider an edge-fed rectangular patch (Fig. 9). Results are compared with those obtained using a published Multiresolution Method of Moment (M RMoM) [10].

The Smith chart in Fig. 10 shows the reflection coefficient S11 against frequency. Without sparsification, a good agreement between the proposed MRSM and the M RMoM is obtained. However at 7.6 GHz, the first resonant frequency, 0.02 GHz discrepancy occurred. This discrepancy may be the result of the approximation of removing the top cover of the box.

Furthermore, in the worst case (argument), with equivalent compression rate of 66% the M RMoM [10] failed to produce a model with good accuracy, while the presented method achieve an error smaller than 5%.
In order to further prove the good performances of the presented method, the analysis of an inset fed patch antenna (Fig. 11) is considered. The results are compared with those presented by a published Multiresolution Method [11].

The effect of sparsification on the accuracy of the solution is considered. Fig. 12 reports the frequency response $|S_{11}|$ for different levels of matrix sparsification.

Fig. 10. Edge-fed patch: reflection coefficient $S_{11}$.

Fig. 11. Inset fed

The effect of sparsification on the accuracy of the solution is considered. Fig. 12 reports the frequency response $|S_{11}|$ for different levels of matrix sparsification.
The results obtained without sparsification are in perfect agreement with the scattering parameters given by the reference [11]. With equivalent accuracy, our approach requires the computation of 82 unknowns while in [11], 109 unknowns have been used. For a large matrix sparsification, the multiresolution method presented by [11] failed to produce a model with good accuracy, while the proposed method achieves an error smaller than 3%. Moreover, the proposed approach permits a considerable improvement of conditioning number. A good conditioning is a key requisite for preserving the accuracy of the inverse matrix under the perturbation introduced by sparsification. With our approach the conditioning number is about 8 while the conditioning number is 19 in [11].

V. Conclusion

A multiresolution source method is achieved for efficient numerical analysis of planar microstrip circuits and antennas. A technique was presented to improve the performance of this method by introducing wavelet trial functions in the representation of the unknown current. The application of the DWT allows the best case to reduce the computation time.

Results obtained for a STEP discontinuity, integrated filters and patch antennas demonstrate that the proposed approach guarantees a high accuracy and decreases required memory and simulation times. The technique described in this paper could be extended to model active circuits, multilayer antennas and arrays.

References