Coupling Characterization and Compensation

Model for Antenna Arrays

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ABSTRACT
Coupling is an important aspect to be considered in the design process of antenna arrays. The real feeding radiated coefficients can be quite different from the theoretical ones because of this effect. In this paper, a characterization and compensation model is presented. This procedure is capable of matching each element of the array. All the parameters of the array coupling model are obtained through the measurement of scattering parameters and radiation patterns of the individual elements. Some practical applications to linear patch arrays are presented.

Keywords - Coupling, matching network, patch antenna, compensation model, active radiation pattern.

INTRODUCTION

According to basic array theory [1], the input impedance and radiation pattern of each element forming the array will be equal to single antenna situation. Under this supposition, the characteristics of every element in the array depend only on its own feeding excitation. But this is not the real behaviour, because the impedance and radiation pattern of the elements in the array depend on the presence of the other radiators, their situation in the array and feeding excitation, the main beam angle and the feeding coefficients of the other elements [2]. Wasylikwski has shown the relation between input impedance and radiated field for Minimum Scattering Antennas (MSA) [3]. Usually, real antennas have been analyzed as MSA with good results [4]. In other cases, that assumption is not so precise and the relation between input coupling matrix and active radiated field is not so clear. Previous published results demonstrate the great influence that coupling effects have in some applications as “finding direction systems”, and the importance of a precise predicting coupling model [5]-[7].

Coupling influence is significantly more noticeable in small arrays, while in large arrays most of the elements present similar conditions: type, position and load. Therefore, the active diagram and impedance are fundamentally the same except for the extreme elements. Conversely, elements active impedance and radiation pattern in small arrays have strong position dependence. In this case, an adequate coupling structure characterization of the elements becomes essential to predict the behaviour of the antenna, as well as a compensation method to restore the proper operation of the antenna array. The input impedance of the array can be characterized with a N ports network model described by the impedance (Z) or scattering (S) matrix (N is the number of elements in the array) [8]. The array radiated field can be expressed as a function of the active radiation pattern of each of its elements [2], [9]
(which contains the coupling effects in the structure). It is also possible to describe the
drated field through Nxk modes, where k is the number of excited modes in each element.
In receiving antennas the model would be the same, defining an active equivalent pattern and
input impedance that depend on the loading circuit.

One of the most important questions in the design of the antenna is if the coupling can be
modelled by a NxN coupling matrix (C) as described in most studies of adaptive antennas
([8] and [10]). Taking into account this idea, in this paper a matrix model is presented for
array antennas connected to a feeding network, which allows a coupling characterization and
compensation of these effects in the array. Although a scattering matrix model was presented
by Lo in [11], it was just applied to receiving antennas arrays and the effect over the global
radiation pattern and input impedance was not studied. The model developed in section II
introduces the concept of fictitious terminals to represent the real radiation of the array
elements, besides the physics accessible terminals only considered in [11]. Thus, a clear
transmission and receiving formulation can be presented considering the different
contributions to the global coupling phenomena. An analysis of the changes over the active
individual impedance, the total reflection coefficient and the real radiation pattern due to
coupling effects is developed. As [2] and [12] proposed, the feeding network contribution is
also taken into account. If the electromagnetic behaviour of each element can be described by
k independent modes, Nxk modes will be needed to define all the array radiation
characteristics independently of the feeding network, feeding distribution or transmission-
reception application [13]. For many radiators, (as printed patches), only one resonant main
radiation mode is enough to describe the radiated field of each element [14]. Under all these
considerations and applying the feeding network parameters (not considered in [15]), a global
coupling matrix C can be obtained to characterize the real radiation pattern and input
impedance of the array. This matrix will also be used to compensate the described effects by
introducing individual matching networks. It is important to point that, through simulations
results or even real radiation patterns and scattering parameters measurements, it is possible
to generate the coupling modelling of the array antenna. In other manuscripts as [8] and [10],
a coupling matrix is calculated from radiation pattern measurements, but it is only applied in
digital range. Likewise, differing from some published papers [16]-[17], this is a general
model independent of the class of radiating element.

In section II we present the theoretical model description, while in section III the model is
applied to three practical cases in linear printed antennas of rectangular patches [18]-[19]. In
these examples, we predict the real behaviour of the antennas due to coupling effects. For the
last antenna, matching compensating networks are also calculated and their validity is probed.

**DESCRIPTION OF MODEL**

**Model for an individual antenna**

A single antenna in free space, excited by an electric current I₀, is characterized by its
input impedance and far field radiation pattern (1):

\[ E₀(θ, φ) = Vₑ₀F₀(θ, φ)e^{-jk₀r}\hat{e}_₀(θ, φ) \]  

(1)

where \( Vₑ₀ \) is a voltage proportional to the input current \( I₀ \), \( F₀(θ, φ) \) the normalized radiation
pattern function, \( \hat{e}_₀(θ, φ) \) the unit polarization vector. It is also possible to work with power
waves, considering the antenna as a two port circuit, corresponding to the physical input
terminal and a non-physical radiating port matched to the intrinsic impedance of the medium.
In that case, the S-parameters equations that describe the antenna are:

\[ b_0 = S_a a_0 + S_e a_{e_0} \]  
\[ b_{e_0} = S_e a_0 + S_{e_0} a_{e_0} \]

where \( a_0 \) represents the incident wave to the antenna input terminals from a generator matched to \( Z_0 \), \( b_0 \) is the reflected wave at the same point, \( b_{e_0} \) is the radiated power wave at the fictitious output terminal (proportional to \( V_{e_0} \) as (4) shows)

\[ b_{e_0} = S_e a_0 = \frac{V_{e_0}}{\sqrt{2\eta_0}} \]

\( a_{e_0} \) represents an external received signal that appears at the output terminal. The physical interpretation of S-parameters introduced in (2)-(3) corresponds with a transmitting and receiving antenna performance. In transmitting operation \( a_{e_0} = 0 \), therefore \( S_a \) represents the input reflection at the physical terminal, while \( S_e \) indicates the transmission radiated coefficient. In a receiving operation \( a_0 = 0 \) if the input terminal is matched to \( Z_0 \). In those conditions, \( S_r \) represents the reception coefficient at the input terminal from an external source, and \( S_d \) is the re-radiated field coefficient due to the interaction with the external \( a_{e_0} \) wave. Using the previous formulation, it is possible to characterize the antenna by means of its basic parameters: radiated field in transmitting operation \( E_0(\theta, \phi) \) (5) and gain \( G_{e_x}(\theta, \phi) \) (6) (related to the antenna efficiency \( \eta \)).

\[ E_0(\theta, \phi) = b_{e_0} \sqrt{2\eta_0} F_0(\theta, \phi) e^{-j\phi_0} \hat{e}_{e_0}(\theta, \phi) \]
\[ G_{e_x}(\theta, \phi) = \eta 4\pi |F_0(\theta, \phi)|^2 = 4\pi \frac{|S_e|^2}{1 - |S_a|^2} |F_0(\theta, \phi)|^2 \]

**Coupling characterization model for an array antenna**

In an array configuration (Fig. 1) the input voltage of each element not only is proportional to the antenna input current \( I_0 \), but also to feeding currents of the other array elements [20]. Therefore, it is necessary to match each new element impedance to get the desired array behaviour. A first approach to this model to take into account these effects has been proposed in [13], [21] and [22], which is fully developed and tested in present manuscript.

![Fig. 1: Array of antennas fed by a distribution network.](image)
As the array is a linear system, the actual field radiated by a group of \( N \) antennas (7) is the sum of the contribution of the active field radiated by each antenna \( \vec{E}_i^{\text{act}}(\theta, \phi) \). For each element in the array, this field is obtained when the rest of the antennas are matched to \( Z_0 \), weighted by the feeding coefficients \( w_i \) [2]. The array field expressed below includes the coupling effects of the radiating elements in the structure.

\[
\vec{E}_{\text{array}}(\theta, \phi) = \sum_{i=1}^{N} w_i \cdot \vec{E}_i^{\text{act}}(\theta, \phi)
\]  

(7)

When the antenna is composed of resonant radiating elements, only one radiating mode is considered. Neglecting the effect of higher modes, the active radiated field of each element \( \vec{E}_i^{\text{act}}(\theta, \phi) \) is obtained as a sum of the contribution \( b_{e_n,i} \) of the main mode radiation field of each element \( \vec{E}_n(\theta, \phi) \), as (8) shows:

\[
\vec{E}_i^{\text{act}}(\theta, \phi) = \sum_{n=1}^{N} b_{e_n,i} \cdot \vec{E}_n(\theta, \phi)
\]  

(8)

Furthermore, as the global antenna is a linear and reciprocal system, it is possible to describe it with a matrix formulation. Fig. 2 depicts a multipole composed of the three main blocks: feeding network, matching coupling compensation network and antennas array block. The matching network will be studied in section 0.

![Fig. 2: Antennas array structure scheme with matching network.](image)

The antennas network consists of \( N \) inputs at the left side representing physical terminals that can be directly measured. The terminals at the right side are the fictitious terminals representing the radiating function. In this case, the antennas multipole can be characterized by a \( 2N \) square matrix (subdivided into 4 square \( N \) dimension matrixes). For similarity to single element model, the square \( N \) dimension \( S_a \) matrix represents the input terminals passive reflection and coupling between the antennas of the array, from a circuital point of view. \( S_e \) is another of the antennas network submatrixes representing the transmission parameters and coupling radiation at the output fictitious terminals. \( S_r \) and \( S_d \) characterize the reception coupling and the scattered field in the array due to an external source. The matrix system relating previous variables is given by:
If matching network is not considered now, power waves with superindex \(^{(1)}\) in Fig. 2 are not applicable: \(a^{(2)}\) is the \(N\) size vector of incident waves to the physical input terminals of the antennas network, while \(b^{(2)}\) is the \(N\)-element vector of reflected waves from the antennas to the feeding network. On the other hand, \(b_e\) is a \(N\) dimension vector containing the coefficients of the main mode radiated field by each of the antennas in the array, different from the desired ones because of coupling effects in the whole structure; \(a_e\) are the received waves from an external incident field into the array, when working as a receiver antenna.

The array feeding network in Fig. 2 makes possible the generation of the feeding coefficients in (7), and the internal coupling effects must also be included in the present model. The feeding network has one input terminal and \(N\) outputs to connect with the \(N\) elements of the array, although it could be generalized to several inputs, for example in multipolarization arrays. The \(N+1\) dimension square matrix that represents it, can be subdivided into the following elements (as shown in Fig. 2): \(S_{00}\) is the reflection coefficient of the feeding network with its outputs matched to ideal \(Z_0\) loads; \(s_q\) represents the \(N\) dimension vector of the transferred power to the \(Z_0\) matched output terminals of the network and finally, \(S_e\) is the \(N\) square dimension coupling matrix corresponding to the output terminals of the feeding network. The feeding network block equations are

\[
\begin{align*}
b_0 &= S_{00}a_0 + s_q^Tb^{(2)} \\
a^{(2)} &= s_q^Ta_0 + S_e^Tb^{(2)}
\end{align*}
\]

where \(a_0\) and \(b_0\) correspond to the incident and reflected waves at the input terminal of the array.

Applying the electromagnetic reciprocity principle, system behaviour will be the same for transmitting or receiving operating mode. Therefore, to simplify the calculation process the array antenna is considered in transmitting mode (\(S_r\) and \(S_d\) are not necessary, and \(a_e\) is the null vector). Results obtained under this supposition will be perfectly applicable to the antenna in receiving operation. Therefore, from (9)-(11) and under previous consideration, it is possible to predict the real radiation pattern of the array with coupling effects by the determination of \(b_e\) as (13) and (14) indicate:

\[
\begin{align*}
b_e &= C_e s_q a_0 \\
C_e &= S_e(I - S_e S_a)^{-1}
\end{align*}
\]

, where \(I\) indicates the unitary matrix. Equation (13) represents a significant result, because it is visible that the power waves at the fictitious terminals, which define the radiating properties of the array elements, are affected by the general coupling matrix \(C_e\) which, as (14) shows, collects the contribution of the coupling effects inside the feed network (\(S_e\)), the array elements network coupling from the circuital point of view (\(S_a\)), and radiating coupling (\(S_e\)). If the designer is trying to obtain a desired radiation pattern, it will be erroneous to implement a feeding network with \(s_q\) as its feeding vector, since the real radiated coefficients \(b_e\) are also affected by the global coupling distortion (\(C_e\)). Therefore, replacing each element of \(b_e\) vector in the individual radiated field equation (5) and adding the contribution of all array elements, the array total radiated field including coupling effects can be expressed as
\[
E_{\text{array}}(\theta, \phi) = d^T(\theta, \phi) \cdot C \cdot s_g a_0 \cdot \sqrt{\frac{2}{\pi}} \cdot e^{-jk\sigma} / r
\]  

(15)

where \(d^T(\theta, \phi)\) is defined as the transposed “steering vector”, which is formed by N elements corresponding to:

\[
d_{\text{n}}(\theta, \phi) = \hat{e}_n(\theta, \phi) \cdot F_n(\theta, \phi) \cdot e^{-jk\sigma r}
\]  

(16)

where \(\hat{e}_n(\theta, \phi)\) and \(F_n(\theta, \phi)\) are the polarization vector and the diagram function of each element in the array (denoted with ‘n’ index). Normally, both parameters are almost equal in all array elements.

Similarly, from (9),(10),(11) and (12) we can predict the reflection coefficient at the array input terminal (17), and the active reflection coefficient of each element inside the array (18). This last reflection can be quite different from one isolated element reflection.

\[
\Gamma_{\text{array}} = \frac{b_0}{a_0} = S_{00} + s_g^T S_a \left( I - S_c S_a \right)^{-1} s_g
\]  

(17)

\[
\left( \Gamma_{\text{act}} \right)_n = \frac{b_n}{a_n} = \frac{\left( S_a \left( I - S_c S_a \right)^{-1} s_g \right)_n}{\left( \left( I - S_c S_a \right)^{-1} s_g \right)_n}
\]  

(18)

**Model coefficients calculation**

At this point, we have presented the expressions for the radiation pattern and input reflection coefficient of the array connected to a feed network considering coupling effects in the structure. But, it is necessary to obtain all the parameters introduced in previous treatise through practical measurements.

In a practical way, all the parameters of the N+1 square feeding network matrix (\(S_{00}, s_g, S_c\)) and the NxN \(S_a\) matrix of the radiating elements, can be easily measured with a vector network analyzer connected to the input and output terminals of the feeding network, and to the physical terminals of the antennas array network. To obtain \(S_e\) matrix, we require the measurement of the individual active radiation patterns of the array elements [2]. Fig. 3 shows the followed spherical test set up.

![Fig. 3: Measurement of the \(S_e\) matrix parameters.](image_url)
Each element of the array is alternatively fed, matching the others with an ideal load $Z_0$. Under these conditions, the field radiated by the structure is measured in far field region (19):

$$\mathbf{E}_i^\text{act}(\mathbf{r}_m) = \sum_{n=1}^{N} b_{e_{z_{z_{i}}}} \cdot \mathbf{E}_n(\mathbf{r}_m) = \sqrt{2\eta} \cdot \sum_{n=1}^{N} b_{e_n} \cdot F_n(\mathbf{r}_{m,n}) \cdot e^{-j\kappa_k|\mathbf{r}_{m,n} - \mathbf{r}_i|} \cdot \hat{e}_n(\mathbf{r}_{m,n})$$  \hspace{1cm} (19)

$\mathbf{E}_i^\text{act}(\mathbf{r}_m)$ denotes the total radiated field by the structure of Fig. 3 in the ‘m’ angle, when ‘i’ element is fed (‘i’ takes values from 1 to N), necessary to $\mathbf{S}_e$ determination; $\mathbf{E}_n$ is the ‘n’ alone element radiated field on its main mode; $\hat{e}_n(\mathbf{r}_{m,n})$ and $F_n(\mathbf{r}_{m,n})$ correspond to the individual polarization vector and radiation pattern function of ‘n’ antenna out of the array structure (usually the same one for all elements in the array can be considered); $\mathbf{r}_m$ corresponds to the position vector of the transmitting horn of the measurement system (there will be as many different vectors as points where $\mathbf{E}_i^\text{act}(\mathbf{r}_n)$ has been measured); $\mathbf{r}_{m,n}$ represents the position of each radiating element in the array under test conditions; $\mathbf{r}_{m,n}$ is the relative position of each one of these elements from the measurement system transmitting horn; and finally $b_{e_{z_{z_{i}}}}$ are the coefficients corresponding to the waves that every element (‘n’) radiates in the test structure when ‘i’ is fed. These coefficients are the components of one $\mathbf{S}_e$ matrix row.

The received power $a_{m,i}$ measured in the test system of Fig. 3 for every ‘m’ point when ‘i’ array element is fed, verifies equation (20):

$$a_{m,i} = \lambda \sum_{i=1}^{N} b_{e_{z_{z_{i}}}} (\hat{e}_n(\mathbf{r}_{m,n}) \cdot \hat{e}_n(\mathbf{r}_{m,n})) \cdot F_n(\mathbf{r}_{m,n}) \cdot F_n(\mathbf{r}_{m,n}) \cdot e^{-j\kappa_k|\mathbf{r}_{m,n} - \mathbf{r}_i|}$$  \hspace{1cm} (20)

and $\lambda$ the wavelength at the measurement frequency. It is clear the dependence on the probe pattern horn ($F_n(\mathbf{r}_{m,n})$) and polarization vector $\hat{e}_n(\mathbf{r}_{m,n})$, which are always a known data in antenna measurement systems. By measuring the radiation pattern function $F_n(\mathbf{r}_{m,n})$ and vector polarization $\hat{e}_n(\mathbf{r}_{m,n})$ of the single isolated elements (usually all of them have the same response), we are able to carry out the diagnostic of the radiated coefficients from (20), where all parameters are known except $b_{e_{z_{z_{i}}}}$. Therefore, after N different measurements as showed in Fig. 3, changing the fed element, it is possible to complete the $\mathbf{S}_e$ matrix extraction process (In Fig. 3 the first element is fed, $i=1$).

As the number of measurement points is larger than the number of unknown parameters ($b_{e_{z_{z_{i}}}}$), we may calculate them from (20) by least squares algorithm. Thus, for each one of the ‘i’ measurements we obtain a set of N $b_{e_{z_{z_{i}}}}$ coefficients, which correspond to one of the row-vectors forming $\mathbf{S}_e$.

At this point we have measured all the necessary parameters to calculate $\mathbf{b}_e$ vector, which allow predicting the real array radiation pattern (15) and reflection coefficient (17).

**Coupling compensation in an array structure**

The model presented in this paper allows to know the coupling effects in the radiated field before manufacturing the entire array. At the same time, these effects should be compensated to have the desired radiation pattern. In this section we present a model to compensate total coupling, with an advantage: we will know the required $\mathbf{s}_g$ vector to get the
compensation before designing the feeding network. The process consists in including individual matching circuits to solve the mismatching of the array elements active impedance. From (13) and (14) we obtain:

\[ \mathbf{s}_g \mathbf{a}_0 = (\mathbf{I} - \mathbf{S}_e \mathbf{S}_a) \mathbf{S}_e^{-1} \mathbf{b}_e. \]  

(21)

We force by means of (22) that coupling from the feeding network does not affect the system. Furthermore, if condition (22) is fulfilled, radiating coupling \( \mathbf{S}_e \) from the array will be taken into account and compensated in the design of the feeding vector \( \mathbf{s}_g \). This is a valid condition for all angles in the array radiation pattern.

\[ \mathbf{S}_a \mathbf{S}_e^{-1} \mathbf{b}_e = 0. \]  

(22)

The way to satisfy (22) is to include a structure which modifies \( \mathbf{S}_a \) and \( \mathbf{S}_e \), as shown in Fig. 2. The new matching network plus the array antennas block, configure an equivalent matched antennas multipole. In this case, the new incident and reflected power waves at the input terminals of the matched antennas array network are \( \mathbf{a}^{(1)} \) and \( \mathbf{b}^{(1)} \). The equivalent \( \mathbf{S}_e \) and \( \mathbf{S}_a \) matrixes will be \( \mathbf{S}^{eq}_a \) (23) and \( \mathbf{S}^{eq}_e \) (24):

\[ \mathbf{S}^{eq}_a = \mathbf{S}_{11} + \mathbf{S}_{21} \mathbf{S}_a (\mathbf{I} - \mathbf{S}_{22} \mathbf{S}_a)^{-1} \mathbf{S}_{21} \]  

(23)

\[ \mathbf{S}^{eq}_e = \mathbf{S}_e (\mathbf{I} - \mathbf{S}_{22} \mathbf{S}_a)^{-1} \mathbf{S}_{21} \]  

(24)

\( \mathbf{S}_{11}, \mathbf{S}_{22} \) and \( \mathbf{S}_{21} \) are diagonal N square matrixes with the scattering parameters \( \mathbf{S}_{11}, \mathbf{S}_{22} \) and \( \mathbf{S}_{21} \) of the individual matching circuits. We have supposed that matching circuits are passive, linear, reciprocal and, as they will not be big size circuits, lossless. By satisfying condition (22) with (23) and (24), it is possible to obtain the individual scattering parameters of the matching circuits to include in the scheme of Fig. 2. If we limit the matching network to N individual matching circuits, the \( \mathbf{S} \) matrixes become diagonal, and the elements in these diagonals satisfy the following equations:

\[ \mathbf{s}^{*}_{22} = (\mathbf{D}[\mathbf{S}^{eq}_e \mathbf{b}_e])^{-1} \cdot \mathbf{S}_a \cdot [\mathbf{S}^{eq}_e \mathbf{b}_e] \]  

(25)

\[ |\mathbf{S}^n_{21}|^2 = 1 - |\mathbf{S}^n_{22}|^2; \quad n=1..N \]  

(26)

\[ \mathbf{S}_{11}^n = - (\mathbf{S}_{22}^n)^* \cdot e^{2j\phi_n}. \]  

(27)

\( \mathbf{s}_{22} \) is a N dimension vector constituted by the single \( \mathbf{S}_{22} \) parameters of the matching circuits (symbol * represents the conjugated value); \( \mathbf{D}[\cdot] \) refers to the diagonal matrix formed by the vector in square brackets as the diagonal of the matrix; \( \mathbf{S}^n_{11}, \mathbf{S}^n_{22} \) and \( \mathbf{S}^n_{21} \) are the individual scattering parameters of ‘n’ element in the matching network; and finally \( \phi_n \) is the phase of \( \mathbf{S}^n_{21} \), free parameter under designer choice. As shown in (25), \( \mathbf{b}_e \) vector is set depending on the desired radiation pattern for the array. In this situation, condition (22) is true and we must design a distribution network with its feeding vector \( \mathbf{s}^{adap}_g \) according to (28), independently of the feeding network couplings \( \mathbf{S}_e \):

\[ \mathbf{s}^{adap}_g = [\mathbf{S}^{eq}_e]^{-1} \mathbf{b}_e. \]  

(28)

Now, all coupling effects are compensated, the array radiation pattern is the desired one, and the individual active impedance of each antenna in the array is matched.
EXPERIMENTAL APPLICATION

We have manufactured three linear arrays with microstrip patches as radiating elements (Fig. 4). The first antenna is a four-element patch array fed with wire probes, the second one is a double +/- 45° polarized four elements array with the same kind of feeding structure, and the third one is made up of eight elements fed by slot coupled line [19]. Their corresponding feeding networks are shown in Fig. 5. All antennas operate in the 3.5 GHz band.

In four elements arrays the validity of the model to characterize coupling effects is only checked, whereas in the third antenna a characterization and compensation operation is done. All necessary parameters have been measured as it was exposed in section 0. By processing these results, we can see in Fig. 6 and Fig. 7 the reflection coefficient and radiation pattern of the linear polarized four elements array.

![Fig. 4. Practical application, antennas array network.](image)

(a) Linear polarized four elements array.  
(b) +/- 45° polarized probe fed patch array.  
(c) slot coupled line fed patch array.

![Fig. 5. Practical application, feeding network.](image)

(a) Linear polarized four elements array.  
(b) +/- 45° polarized four elements array.  
(c) Linear polarized eight elements array.

![Fig. 6: Reflection coefficient of linear polarized four elements array.](image)
In Fig. 6, it is visible the degradation of input reflection coefficient comparing the feeding network with ideal $Z_0$ loads at the output terminals and the complete structure (feeding network with radiating elements). The coupling model offers a satisfactory approximation to array reflection. To follow with, in Fig. 7 we have represented the model prediction and the measured radiation pattern, considering a uniform module and phase excitation. $E_{cp}$ represents the copolar component of the electric field. There is a good agreement in the level of main and secondary lobes. In the measured pattern a null filling appears between the main and first side lobes that the model is not able to predict. The explanation is that we have only considered a main radiating mode, and we have not computed the effect of higher modes. We notice that the measured radiation pattern is quite close to the model.

Fig. 8 and Fig. 9 compare the input reflection coefficient of the independent feeding network for $\pm 45^\circ$ polarized antennas with ideal $Z_0$ loads, the complete $45^\circ$ polarization array antenna (feeding network with radiating elements) and the model estimation. It is clear the degradation in the array reflection response for both polarizations, well predicted by the model.

Fig. 7: Radiation pattern of linear polarized four elements array.

Fig. 8: Reflection coefficient of $\pm 45^\circ$ polarized four elements array.
Fig. 9: Reflection coefficient of - 45º polarized four elements array.

Fig. 10 and Fig. 11 represent the copolar component of the 45º polarization antenna comparing the model estimation with the real measurement. There is a noteworthy similarity between measured and modelled radiation pattern over the main and first side lobes. The results are quite satisfactory above all in copolar component.

With the previous prototypes we have demonstrated that the proposed model is suitable to be applied. In the third manufactured prototype we have realised a more detailed analysis of the effects, and we have also compensated them. To get a side lobe level of -25 dB referenced to main lobe, the desired radiating coefficients in amplitude are as follows:

\[-7.5,-4.5,-1.5,0,0,-1.5,-4.5,-7.5\] (dB) . (29)
The phase is uniform. We have measured all the coupling parameters as indicated in section 0, and calculated the input reflection coefficient and radiation pattern of the array. We have compared these measurements with the presented model.

As we did for four elements array, we have represented in Fig. 12 the reflection coefficient of the feeding network with $Z_0$ end terminals loads, compared to the reflection of the complete array. It is clear that a strong mismatch is produced. Likewise, we have represented the estimated input reflection given by the model, which is quite close to the real measured value.

![Reflection Coefficient](image)

**Fig. 12:** Reflection coefficient of eight elements array.

To clearly understand what happened in the array when the feeding network has been connected to radiating elements and all of them are fed at the same time, we can see in Fig. 13 the evolution of the impedance of each element out and inside the array. We have represented the passive and active reflection of each element in the array. We call passive reflection the one measured with the patch in its position in the array structure, but without feeding the array with the distribution network (the rest of patches have $Z_0$ loads). Under these conditions, reflection is very low, better than $-22$ dB for all patches and frequencies. On the other hand, when the feeding network is joined to the patches, the individual reflection for each one of them becomes appreciably worse (active impedance).

![Impedance Degradation](image)

**Fig. 13:** Individual array elements reflection. Impedance degradation caused by coupling effects.

The radiation pattern suffers the consequences of mismatching due to coupling effects, as we appreciate in Fig. 14. We have represented the measured and modelled prediction pattern, as well as the theoretical radiation pattern that the array should have under ideal conditions.
(29). The level of first side lobe has increased from under -30 dB (theoretical pattern) to -20 dB (measurement). Likewise, there is a noteworthy agreement between measured and model radiation pattern, especially over the main and first side lobes. As we indicated in four elements arrays, we have only considered the main mode in the model.

If we want to achieve the desired pattern and a lower input reflection coefficient, we have to apply the compensation method described in section 0. We extract the individual matching scattering parameters from (25)-(27), doing $b_e$ vector equal to (29) in module and uniform phase. As we had previously measured $S_a$ with a network analyzer, and $S_e$ with the radiation pattern procedure described in section 0, we have all necessary data to implement the matching network. The basic structure implemented to reproduce the calculated scattering parameters is very simple (Fig. 15), composed of three sections with different widths.

We have optimized the physical dimensions to the calculated scattering parameters values using ADS Agilent tool. Due to the symmetric amplitude distribution of the array (29), four different symmetrically placed circuits are necessary to complete the process. Each matching circuit is connected to its corresponding input antenna terminal in the array as depicted in Fig. 15. As an example, matching circuit dimensions corresponding to element 1 and 8 (the same because of the symmetry) are: $W_1=10$ mm, $W_2=4.23$ mm, $W_3=1.4$ mm, $L_1=1.6$ mm, $L_2=$
21.32 mm, \(L_3 = 32\) mm. Likewise, calculated and implemented S-parameters are represented in Fig. 16.

As it was indicated in section 0, we should now design a feeding network with a distribution according to equation (28), connecting each output terminal to its matching circuit. This feeding network, added to the matching circuits, will compensate all coupling effects in the structure. Curiously, the \(s^{\text{adapt}}\) vector from (28) is very similar to the feeding vector that the original network had, so we have decided to use it. The results are quite good as Fig. 17 and Fig. 18 demonstrate.
Matching circuits and feeding network have slight differences from the calculated parameters, but they are accurate enough to compensate the coupling effects and active impedance mismatching. The new measured radiation pattern is the desired one (Fig. 17) and the input reflection coefficient in the array is nearly the same as the feeding network value (Fig. 18). This last point is quite obvious because the new active reflection coefficient of each antenna in the array is -45 dB, so they are perfectly matched considering coupling effects. Now, every antenna in the array is similar to an ideal $Z_0$ load, therefore we have reproduced the conditions of the feeding network measurement.

CONCLUSIONS

A general model to characterize and compensate coupling effects in an array of antennas has been presented. This model is based on the linearity properties of the array elements and takes into account the main radiation mode excited in every element of the array. All necessary parameters to characterize coupling effects in the array, including the active radiation pattern of individual antennas, are measured. These effects can be compensated, both in radiation pattern and input reflection, by introducing individual matching circuits. We have demonstrated the validity of this model with three different patch array structures, where we have been able to predict very satisfactorily the coupling behaviour and to mend appeared effects, even considering only the main excited mode in the patches.

Next steps in this research will be pointed in large structures, as radial line planar antennas. Normally the most of the elements have similar conditions, so the number of measurements can be notably decreased and computing time will be reasonable. On the other hand, the benefits of using this model in preliminary steps of an array design can save a considerable amount of time, predicting the effects of coupling between radiating elements. All necessary parameters could be obtained by electromagnetic modellers, and applied to the proposed model before manufacturing the antenna.

REFERENCES


