ELECTROMAGNETIC WAVE POSITIVE PROPAGATION IN DOUBLY MODULATED TWO-DIMENSIONALLY PERIODIC MEDIA

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Abstract

The paper presented a new periodic model, Doubly modulated Two-dimensionally (D2D) periodic media. Under the condition of positive propagation, characteristic equations in the dyad form for TE waves and TM waves are derived. By satisfying the sufficient and necessary condition for existence of the nontrivial solution of characteristic equations, the dispersion relations of TE waves and TM waves in D2D periodic media are obtained. Moreover, interactions between space harmonics were also studied. The research on D2D model in the paper is not only the development of the electromagnetic theory of periodic media, but also useful for actual engineering applications.

1. INTRODUCTION

Photonic bandgap technology [1] and periodic dielectric structure [2] are already used widely in novel microwave and millimeter-wave devices in recent years. Actually, the photonic crystals or photonic bandgap materials are periodic media. Since they can produce frequency band gaps which suitable to guide electromagnetic energy, periodic media have been received significant attention. Typical methods to study periodic media are coupled-modes theory [3-5], scattering matrix theory [6], modes-matched method [7] etc. A general approach is presented for two-dimensionally periodic (2DP) media [8].

In this paper, a novel more complex periodic media model has been presented. What makes the periodic media different from others is that they are doubly modulated. Under the condition of positive propagation, characteristic equations in the dyad form for TE waves and TM waves in D2D are derived by rigorous electromagnetic theory. The dispersion relation of the positive electromagnetic waves in D2D periodic media is given on the basis of numerical simulation. Moreover, space harmonics interactions in Brillouin diagram of D2D media are compared with those of 2DP media. The possible application of the novel D2D media is also pointed out.

2. CHARACTERISTIC EQUATION FOR POSITIVE WAVES

The permittivity distribution of the new D2D periodic media model is

\[ \varepsilon(x, y) = \varepsilon_0 [1 + D(x, y)\varepsilon'(x, y)] \]  

where

\[ D(x, y) = D_0 \cos(2\pi x) \cos(2\pi y) \]
(2) and (3) are periodic with periods \(a, A\) and \(b, B\) in \(x\)- and \(y\)-direction, respectively, moreover \(a = A/p\) and \(b = B/q\), \(p, q\) belonging to integers. Namely, \(A = pa\) and \(B = qb\) are common periods for both functions \(D(x, y)\) and \(\varepsilon'(x, y)\). However, its distribution is uniform in \(z\)-direction.

According to Floquet theorem and properties of a complete set of the modal solutions, the characteristic equation has been deduced on the basis of [8]

\[
(k_{ij} \cdot \tilde{k}_{ij} I - \tilde{k}_{ij} \tilde{k}_{ij} )\tilde{\varepsilon}_{ij} = k_0^2 \tilde{\varepsilon}_{ij} + k_0^2 I \sum_{m,n=-\infty}^{\infty} \iint \limits_S D(x,y)\varepsilon'(x,y)\psi_{mn} \psi_{mn}^- dxdy \tag{4} \]

\[
\psi_{mn} = (AB)^{-\frac{1}{2}} \exp(-jk_{mn}x - jk_{mn}y) \tag{5} \]

\(I\) is a three-order unit matrix and \(k_{mn} = k_x + 2m\pi/A, \ k_{yn} = k_y + 2n\pi / B\). \(\sum\) stands for the summation over all integers \(m, n\), and \(\int\) for the integral over the area \(AB\). To simplicity, two operators are introduced.

\[
T_{ij} = [(k_{0}^2 - k_{ij}^2)I + \tilde{k}_{ij} \tilde{k}_{ij}] \tag{6} \]

\[
D_{mn} = \langle \psi_{mn} | D(x,y)\varepsilon', (x,y) | \psi_{mn}^- \rangle >_{A\times B} \tag{7} \]

Substituting (6) and (7) into (4) results in

\[
T_{ij} \tilde{\varepsilon}_{ij} + k_0^2 I \sum_{m,n=-\infty}^{\infty} D_{mn} \tilde{\varepsilon}_{mn} = 0 \tag{8} \]

(8) is the characteristic equation of D2D periodic media. When the positive electromagnetic wave propagated in D2D periodic media, namely, \(k_z = 0\). (8) is turned into two independent control equations as follows

\[
T_{ij} \tilde{\varepsilon}_{ij} + k_0^2 I \sum_{m,n=-\infty}^{\infty} D_{mn} \tilde{\varepsilon}_{mn} = 0 \tag{9} \]

\[
T_{ij} \tilde{\varepsilon}_{ij} + k_0^2 I \sum_{m,n=-\infty}^{\infty} D_{mn} \tilde{\varepsilon}_{mn} = 0 \tag{10} \]

(9) is for TE mode and (10) for TM mode. Here \(T_{ij}\) and \(T_{ij}\) are transverse, longitudinal part of operator \(T_{ij}\) respectively. In (9), \(I\) is two-order unit matrix, \(\tilde{\varepsilon}_{mn} = \tilde{x}\tilde{\varepsilon}_{xmn} + \tilde{y}\tilde{\varepsilon}_{ymn}, \ \tilde{\varepsilon}_{mn}\) is transverse and longitudinal electric field amplitude value respectively. Nontrivial solution exists only if the determinant of coefficient matrix of equation (9) and (10) equal zero. That is, the determinant of coefficient matrix of equation (9) equals zero leads to dispersion equation of TE modes, while the same way to (10) for TM modes.

3. NUMERICAL ANALYSIS AND DISCUSSION

A D2D periodic media with its unit cell shown in Fig.1 is taken as an example. Both
$D(x,y)$ and $\varepsilon_r(x,y)$ are piecewise uniform that

\[
\varepsilon_r = \begin{cases} 
0.1 & \text{for } 0 \leq x \leq a_1 \text{ and } 0 \leq y \leq b_1, \\
0 & \text{for } a_1 \leq x \leq a \text{ or } b_1 \leq y \leq b 
\end{cases}
\]

\[
D = \begin{cases} 
1 & \text{for } 0 \leq x \leq u a \text{ and } 0 \leq y \leq v b, \\
0 & \text{for } u a \leq x \leq A \text{ or } u b \leq y \leq B 
\end{cases}
\]

and $p = q = 11$, radius of unperturbed circle $R = (k_0^2 \varepsilon_{\text{eff}})^{1/2} = 1.18A$. When $k_z = 0$, the Brillouin diagram for TE mode is shown in Fig. 2 in which twelve space harmonics such as $(1,0)$, $(1,-1)$, $(0,1)$, $(0,0)$, $(0,-1)$, $(0,-2)$, $(-1,1)$, $(-1,0)$, $(-1,-1)$, $(-1,-2)$, $(-2,0)$, $(-2,-1)$ are considered. That for TM mode is similar and not shown.

In Fig.2, the thin solid lines stand for un-turbulent circle with radius $r$ of uniform dielectric while the heavy dot points stand for the actual Floquet mode. It can be easily found that not only the interactions between space harmonics are much stronger in D2D than those in 2DP at the same condition, but also the numbers of stop-bands in D2D are much more than those in 2DP. All these will result in more abundant application.

The propagation characteristics of electromagnetic wave in stop-band of D2D periodic media are shown separately for clarity. Fig.3 shows a stop-band of Brillouin diagram for TM mode, while Fig.4 depicts a stop-band of Brillouin diagram for TE mode. In each stop-band,
the two part of transverse wave vector can’t be real. That is, \( k_x \) or \( k_y \) may be complex. In Fig.3, \( k_y \) is real, but \( k_x \) is complex with its real part 0.5. This means wave propagation in y-direction and attenuation in x-direction. So in this situation electromagnetic wave can’t propagate in x-direction. Fig.4 is contrary with Fig. 3. From the two diagrams it can be seen that under the same condition, the max imaginary part of \( k_x \) in Fig.3 is 0.024, while the max imaginary part of \( k_y \) in Fig.4 is 0.015. It showed that TM mode had much greater attenuation than TE mode.

4. CONCLUSIONS

A new doubly modulated (D2D) periodic media model has been stated in this paper. The propagation characteristics of electromagnetic wave in D2D periodic media when \( k_z \) equals zero have obtained. Other electromagnetic property such as its dispersion diagrams and stop-bands have also analyzed through a simple numerical illustration. The D2D modal discussed in the paper is not only the development of the electromagnetic theory of periodic media, but also useful for actual engineering applications.

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REFERENCES